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Collapsing stage of "bosonic matter"

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Abstract

We prove rigorously that for "bosonic matter", if deflation occurs upon collapse as more and more such matter is put together, then for a non-vanishing probability of having the negatively charged particles, with Coulomb interactions, within a sphere of radius R, the latter *necessarily* cannot decrease faster than $N^{-1/3}$ for large N, where N denotes the number of the negatively charged particles. This is in clear distinction with matter (i.e., matter with the exclusion principle) which inflates and R necessarily increases not any slower than $N^{1/3}$ for large N.

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The astonishment as to why matter occupies so large a volume and its connection to the Pauli exclusion principle was clearly expressed in words addressed by Ehrenfest to Pauli in 1931 on the occasion of the Lorentz medal (cf. [1]) to this effect: "We take a piece of metal, or a stone. When we think about it, we are astonished that this quantity of matter should occupy so large a volume". He went on stating that the Pauli exclusion principle is the reason: "Answer: only the Pauli principle, no two electrons in the same state". In regard to this, we have recently shown [2] that for a non-vanishing probability of having electrons in matter, with Coulomb interactions, within a sphere of radius R, the latter necessarily grows not any slower than $N^{1/3}$ for large N, where N denotes the number of electrons. This conclusion is based on a derived inequality [2] relating the probability for the electrons to lie within such a sphere, the volume v_R of the latter and the number N of electrons:

$$\mathtt{Prob}\big[|\mathbf{x}_1| \leqslant R, \ldots, |\mathbf{x}_N| \leqslant R\big]\left(\frac{N}{v_R}\right)^{2/5} < \left(\frac{1}{a_0^3}\right)^{2/5} 1.846\big[1 + Z^{2/3}\big]^{6/5}, \tag{1}$$

where $a_0 = \hbar^2/me^2$ is the Bohr radius, and $Z|e|$ corresponds to the nucleus in matter carrying the largest positive charge. The above statement follows by noting from (1) that for a non-vanishing probability of having the electrons within the sphere, the corresponding volume v_R grows not any slower than the first power of N for $N \to \infty$, since otherwise the left-hand side of (1) would go to infinity and would be in contradiction with the finite upper bound on its right-hand side. We also note that N/v_R gives an average density, and one may also infer from (1) that the infinite density limit $N/v_R \to \infty$ does not occur, as the probability on the left-hand side of (1) necessarily goes to zero in such a limit.

The Hamiltonian in question is taken to be the N-electron one

$$H = \sum_{i=1}^{N} \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j}^{N} \frac{e^2}{|\mathbf{x}_i - \mathbf{x}_j|} - \sum_{i=1}^{N}\sum_{j=1}^{k} \frac{Z_j e^2}{|\mathbf{x}_i - \mathbf{R}_j|} + \sum_{i<j}^{k} \frac{Z_i Z_j e^2}{|\mathbf{R}_i - \mathbf{R}_j|}, \tag{2}$$

where $\mathbf{x}_i$, $\mathbf{R}_j$ correspond, respectively, to positions of electrons and nuclei. We have also considered neutral matter $\sum_{i=1}^{k} Z_i = N$.

What conclusion can be drawn about matter if the Pauli exclusion principle is not invoked?—that is regarding "bosonic

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matter” [1,3,4]. Here we recall the drastic difference between matter (with the exclusion principle) and “bosonic matter” is that the ground-state energy E_N for the former $-E_N \sim N$ [5, 6], while for the latter [1,3,4,7] $-E_N \sim N^\alpha$ with $\alpha > 1$. And such a power law behavior with $\alpha > 1$ implies instability as the formation of a single system consisting of $(2N + 2N)$ particles is favored over two separate systems brought together each consisting of $(N + N)$ particles, and the energy released upon the collapse of the two systems into one, being proportional to $[(2N)^\alpha - 2(N)^\alpha]$ will be overwhelmingly large for realistic large N, e.g., $N \sim 10^{23}$. In regard to such a collapse Dyson states [1]: “[Bosonic] matter in bulk would collapse into a condensed high-density phase. The assembly of any two macroscopic objects would release energy comparable to that of an atomic bomb... Matter without the exclusion principle is unstable”.

We prove rigorously that if deflation does occur for “bosonic matter”, upon collapse, as more and more such matter is put together, then for a non-vanishing probability of having the negatively charged particles within a sphere of radius R, the latter *necessarily* cannot decrease faster than $N^{-1/3}$ for large N. To this end, we define the particle density of N (spin 0) bosons:

$$\rho(\mathbf{x}) = N \int \mathrm{d}^3\mathbf{x}_2 \cdots \mathrm{d}^3\mathbf{x}_N \left|\phi(\mathbf{x}, \mathbf{x}_2, \ldots, \mathbf{x}_N)\right|^2 \tag{3}$$

and $\int \mathrm{d}^3\mathbf{x}\, \rho(\mathbf{x}) = N$, ϕ denotes a normalized state giving a strictly negative expectation value of the Hamiltonian, i.e.,

$$-\epsilon_N[m] \leqslant \langle\phi|H|\phi\rangle < 0, \tag{4}$$

where $-\epsilon_N[m] = E_N < 0$ is the ground-state energy emphasizing its dependence on m.

To establish the statement made above, we need [2] upper and lower bounds to the expectation value of the kinetic energy operator

$$T \equiv \langle\phi| \sum_{i=1}^N \frac{\mathbf{p}_i^2}{2m} |\phi\rangle. \tag{5}$$

To the above end, we rewrite $|\phi\rangle = |\phi(m)\rangle$, emphasizing its dependence on the mass m. Since $|\phi(m/2)\rangle$ cannot lead for $\langle\phi(m/2)|H|\phi(m/2)\rangle$ a numerical value lower than $-\epsilon_N[m]$, we have $-\epsilon_N[m] \leqslant \langle\phi(m/2)|H|\phi(m/2)\rangle$. Accordingly, if we denote the interaction part in (2) by V, we have

$$-\epsilon_N[2m] \leqslant \left\langle\phi(m)\right|\frac{T}{2} + V\left|\phi(m)\right\rangle \tag{6}$$

and hence we have from the extreme right-hand side of the inequality (4)

$$T \leqslant 2\epsilon_N[2m]. \tag{7}$$

A lower bound for T was derived in [8]. The basic idea in that derivation is to consider an effective interaction of the form $g(\mathbf{x}) = 4\rho(\mathbf{x})/(3\int \mathrm{d}^3\mathbf{x}\, \rho^2(\mathbf{x}))$, coupled with the way of counting the number of eigenvalues, in the manner of Schwinger [9], of the effective Hamiltonian $\sum_{i=1}^N [\mathbf{p}_i^2/2m - g(\mathbf{x}_i)]$. This gives the lower bound [8]

$$\frac{3\hbar^2}{2mN^{1/3}}\left(\frac{\pi}{2}\right)^{2/3}\frac{1}{1+\varepsilon}\left(\int \mathrm{d}^3\mathbf{x}\, \rho^2(\mathbf{x})\right)^{2/3} \leqslant T \tag{8}$$

for any $\varepsilon > 0$ which may be taken as small as we please.

The lower bound expression obtained in [8] for $-\epsilon_N[m]$ may be now used to derive from (7) and (8) the basic relations

$$\begin{aligned}&\frac{3\hbar^2}{2mN^{1/3}}\left(\frac{\pi}{2}\right)^{2/3}\frac{1}{1+\varepsilon}\left(\int \mathrm{d}^3\mathbf{x}\, \rho^2(\mathbf{x})\right)^{2/3}\\ &\quad \leqslant T < 3.78\left(\frac{me^4}{\hbar^2}\right)N^{5/3}\left[1 + \sum_{i=1}^k \frac{Z_i^2}{N}\right]^{4/3}.\end{aligned} \tag{9}$$

For the probability of the N negatively charged particles to lie within a sphere of radius R, we have

$$\begin{aligned}&\mathtt{Prob}\big[|\mathbf{x}_1| \leqslant R, \ldots, |\mathbf{x}_N| \leqslant R\big]\\ &\quad \leqslant \mathtt{Prob}\big[|\mathbf{x}_1| \leqslant R\big]\\ &\quad = \frac{1}{N}\int \mathrm{d}^3\mathbf{x}\, \rho(\mathbf{x})\mathcal{X}_R(\mathbf{x})\\ &\quad \leqslant \frac{1}{N}\left(\int \mathrm{d}^3\mathbf{x}\, \rho^2(\mathbf{x})\right)^{1/2}(v_R)^{1/2},\end{aligned} \tag{10}$$

where $\mathcal{X}_R(\mathbf{x}) = 1$ if $|\mathbf{x}| \leqslant R$, and $= 0$ otherwise. In writing the last inequality in (10) we have used the Cauchy–Schwarz inequality and that $(\mathcal{X}_R(\mathbf{x}))^2 = (\mathcal{X}_R(\mathbf{x}))$, $v_R = 4\pi R^3/3$.

From (9), (10), we then have the explicit inequality

$$\begin{aligned}&\mathtt{Prob}\big[|\mathbf{x}_1| \leqslant R, \ldots, |\mathbf{x}_N| \leqslant R\big]\frac{1}{(v_R N)^{1/2}}\\ &\quad < \left(\frac{1}{a_0^3}\right)^{1/2} 1.61[1+Z].\end{aligned} \tag{11}$$

From this inequality we may infer the inescapable fact that if deflation of “bosonic matter” occurs, upon collapse, then for a non-vanishing probability of having the N negatively charged particles within a sphere of radius R, the corresponding volume, necessarily, cannot shrink faster than $1/N$ for $N \to \infty$, since otherwise the left-hand side of (11) would go to infinity and would be in contradiction with the finite upper bound on its right-hand side, thus establishing the above stated result. We note that the inequality in (11) is sufficient to reach such a conclusion but cannot establish the actual deflation of such matter. This formidable problem will be attempted in a future report. Methods similar to the ones developed above have been used to study the localizability and stability of other quantum mechanical systems as well [10].

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