Approximation by Lagrange Splines

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Abstract. Polynomials and their smooth piecewise analogs known as splines are used as the basic means of approximation in nearly all areas of numerical analysis. For this reason, the representation and evaluation of polynomials and splines is a fundamental topic in numerical analysis. We will discuss this topic in the context of local spline interpolation, the simplest and certainly the most widely used technique for obtaining spline approximation. One central point of this paper is a generalization of Horner's rule for the simultaneous evaluation of the interpolating polynomial and its derivatives. Such an algorithm is usually not found in standard textbooks on numerical analysis.
We will study the simplest piecewise polynomial approximations known as Lagrange interpolating splines in detail. Using a very simple approach we show how to obtain smooth analogues of Lagrange splines which only approximate the data while still providing the same order of approximation as Lagrange interpolating splines. Such functions are usually called quasi-interpolants. We then study the commonly used Lagrange splines, such as piecewise cubic and piecewise quadratic Lagrange polynomials in detail. Relations between discrete polynomial splines and Lagrange splines are investigated including a generalization of Marsden's identity [16].