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$\bar{p}D$ atoms in realistic potentials

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Abstract

The $\bar{p}D$ atoms are studied in various realistic, popular $\bar{N}N$ potentials. The small energy shifts and decay widths of the atoms, which stem from the short-ranged strong interactions between the antiproton and deuteron, are evaluated in a well-established, accurate approach based on the Sturmian functions. The investigation reveals that none of the employed potentials, which reproduce the $\bar{N}N$ scattering data quite well, is able to reproduce the experimental data of the energy shifts of the $2p$ $\bar{p}D$ atomic states. The energy shifts of the $2p$ $\bar{p}D$ atomic states are very sensitive to the $\bar{N}N$ strong interactions, hence the investigation of the $\bar{p}D$ atoms is expected to provide a good platform for refining the $\bar{N}N$ interaction, especially at zero energy.
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1. Introduction

The second simplest antiprotonic atom is the antiprotonic deuteron atom $\bar{p}D$, consisting of an antiproton and a deuteron bound mainly by the Coulomb interaction but distorted by the short range strong interaction. The study of the $\bar{p}D$ atom is much later and less successful than for other exotic atoms like the protonium and pionium. Experiments were carried out at LEAR just in very recent years to study the properties of the $\bar{p}D$ atom [1,2]. Even prior to the experiments some theoretical works [3–5] had been carried out to study the $\bar{p}D$ atomic states in simplified $\bar{p}D$ interactions. Recently, a theoretical work [6] proposed a mechanism explaining the unexpected behavior, of the scattering lengths of $\bar{N}N$ and $\bar{p}D$ system, that the imaginary part of the scattering length does not increase with the size of the nucleus.

In the theoretical sector, one needs to overcome at least two difficulties in the study of the $\bar{p}D$ atom. First, the interaction between the antiproton and the deuteron core should be derived from realistic $\bar{N}N$ interactions, for example, the Paris $\bar{N}N$ potentials [7–9], the Dover–Richard $\bar{N}N$ potentials I (DR1) and II (DR2) [10,11], and the Kohno–Weise $\bar{N}N$ potential [12]. Even if a reliable $\bar{p}D$ interaction is in hands, the accurate evaluation of the energy shifts and decay widths (stemming for the strong $\bar{p}D$ interactions) and especially of the nuclear force distorted wave function of the atom is still a challenge. It should be pointed out that the methods employed in the works [3–5] are not accurate enough for evaluating the wave functions of the $\bar{p}D$ atoms.

In the present work we study the $\bar{p}D$ atom problem employing a properly adapted numerical method based on Sturmian functions [13]. The method accounts for both the strong *short* range nuclear potential (local and non-local) and the *long* range Coulomb force and provides directly the wave function of the $\bar{p}D$ system with complex eigenvalues $E = E_R - i\frac{\Gamma}{2}$. The protonium and pionium problems have been successfully investigated [14,15] in the numerical approach. The numerical method

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is much more powerful, accurate and much easier to use than all other methods applied to the exotic atom problem in history. The $\bar{p}D$ interactions in the work are derived from various realistic $\bar{N}N$ potential, which is state-dependent. The work is organized as follows. The $\bar{p}D$ interactions are expressed in Section 2 in terms of the $\bar{N}N$ interactions. In Section 3 the energy shifts and decay widths of the $1s$ and $2p$ $\bar{p}D$ atomic states are evaluated. Discussions and conclusions are given in Section 3, too.

2. $\bar{p}D$ interactions in terms of $\bar{N}N$ potentials

We start from the Schrödinger equation of the antiproton–deuteron system in coordinate space

$$\left(\frac{P_\rho^2}{2M_\rho} + \frac{P_\lambda^2}{2M_\lambda} + V_{12}(\vec{r}_2 - \vec{r}_1) + V_{13}(\vec{r}_3 - \vec{r}_1)\right.$$
$$\left. + V_{23}(\vec{r}_3 - \vec{r}_2)\right)\Psi(\vec{\lambda}, \vec{\rho}) = E\Psi(\vec{\lambda}, \vec{\rho}) \tag{1}$$

where $\vec{\lambda}$ and $\vec{\rho}$ are the Jacobi coordinates of the system, defined as

$$\vec{\lambda} = \vec{r}_3 - \frac{\vec{r}_1 + \vec{r}_2}{2}, \qquad \vec{\rho} = \vec{r}_2 - \vec{r}_1, \tag{2}$$

$M_\rho = M/2$ and $M_\lambda = 2M/3$ are the reduced masses. Here we have assigned, for simplicity, the proton and neutron the same mass M. Eq. (1) can be expressed in the form, where the strong interaction is expressed in the isospin basis,

$$\left(\frac{P_\rho^2}{2M_\rho} + \frac{P_\lambda^2}{2M_\lambda} + V_S + V_C\right)\Psi(\vec{\lambda}, \vec{\rho}) = E\Psi(\vec{\lambda}, \vec{\rho}) \tag{3}$$

where V_S and V_C stand for the nuclear interaction and Coulomb force, respectively, and take the forms

$$V_S = V_{NN}^0(\vec{r}_2 - \vec{r}_1) + \frac{1}{4}\left[V_{\bar{N}N}^0(\vec{r}_3 - \vec{r}_1) + V_{\bar{N}N}^0(\vec{r}_3 - \vec{r}_2)\right]$$
$$+ \frac{3}{4}\left[V_{\bar{N}N}^1(\vec{r}_3 - \vec{r}_1) + V_{\bar{N}N}^1(\vec{r}_3 - \vec{r}_2)\right], \tag{4}$$

$$V_C = \frac{1}{2}\left[V_C(\vec{r}_3 - \vec{r}_1) + V_C(\vec{r}_3 - \vec{r}_2)\right] \tag{5}$$

V^0 and V^1 in Eq. (4) are the isospin 0 and 1 nuclear interactions, respectively. Note that we have assigned $\vec{r}_{12}$ as the relative coordinate of the deuteron core.

One may express the interactions V_C and V_S in Eqs. (4) and (5) in terms of the interactions of certain $\bar{N}N$ states. In the $|JMLS\rangle$ basis of the $\bar{p}D$ states

$$|JMLS\rangle = \left|(L_\rho \otimes L_\lambda)_L \otimes (S_{12} \otimes S_3)_S\right\rangle_{JM} \tag{6}$$

we derive

$$\left(H_0 + W_C(\lambda, \rho) + V_{NN}^0(\rho) + W_S(\lambda, \rho)\right)\Psi(\lambda, \rho)$$
$$= E\Psi(\lambda, \rho) \tag{7}$$

with

$$H_0 = \frac{P_\rho^2}{2M_\rho} + \frac{P_\lambda^2}{2M_\lambda} \tag{8}$$

W_C and W_S in Eq. (7) are respectively the Coulomb force and strong interaction between the antiproton and deuteron, and V_{NN}^0 the interaction between the proton and neutron in the deuteron core. W_C and W_S are derived explicitly as

$$W_C(\lambda, \rho) = \frac{1}{2}\int_{-1}^{1} dx\, V_C(r_{13}), \tag{9}$$

$$W_S(\lambda, \rho)$$
$$= \frac{1}{2}\int_{-1}^{1} dx \sum_{Q, Q'} \langle P|Q\rangle\langle Q|V_{\bar{N}N}(\vec{r}_{13})|Q'\rangle\langle Q'|P'\rangle \tag{10}$$

with

$$V_{\bar{N}N}(\vec{r}_{13}) = \frac{1}{2}V_{\bar{N}N}^0(\vec{r}_{13}) + \frac{3}{2}V_{\bar{N}N}^1(\vec{r}_{13}), \tag{11}$$

$$r_{13} \equiv |\vec{r}_1 - \vec{r}_3| = \left(\lambda^2 + \frac{\rho^2}{4} - \lambda\rho x\right)^{1/2} \tag{12}$$

where $x = \cos\theta$ with θ being the angle between $\vec{\lambda}$ and $\vec{\rho}$. In Eq. (10) $|P\rangle \equiv |JMLS\rangle$ and $|P'\rangle \equiv |JML'S\rangle$ are as defined in Eq. (6) while the states $|Q\rangle$ and $|Q'\rangle$ are

$$|Q\rangle = \left|(L_\sigma \otimes S_{13})_{J_\sigma} \otimes (L_\gamma \otimes S_2)_{J_\gamma}\right\rangle_{JM}, \tag{13}$$

$$|Q'\rangle = \left|(L'_\sigma \otimes S_{13})_{J_\sigma} \otimes (L_\gamma \otimes S_2)_{J_\gamma}\right\rangle_{JM}. \tag{14}$$

Here $\vec{\sigma}$ and $\vec{\gamma}$ are also the Jacobi coordinates of the system, defined as

$$\vec{\gamma} = \vec{r}_2 - \frac{\vec{r}_1 + \vec{r}_3}{2}, \qquad \vec{\sigma} = \vec{r}_3 - \vec{r}_1. \tag{15}$$

So defined the states $|Q\rangle$ and $|Q'\rangle$ is based on the consideration that the $\bar{N}N$ interactions can be easily expressed in the $|J_\sigma M_\sigma L_\sigma S_{13}\rangle$ basis of the $\bar{N}N$ states. Note that $\langle P|Q\rangle$ depends on not only the quantum numbers of the states $|P\rangle$ and $|Q\rangle$, but also λ, ρ and the angle θ between $\vec{\lambda}$ and $\vec{\rho}$ resulting from the projection of the orbital angular momenta between different Jacobi coordinates. We listed the integral kernels in Eq. (10), $\sum_{Q,Q'}\langle P|Q\rangle\langle Q|V(\vec{r}_{13})|Q'\rangle\langle Q'|P'\rangle$, for the lowest $\bar{p}D$ states in the approximation that the deuteron core is assumed in the S-state, as follows:

$$|P\rangle = |P'\rangle = |^2S_{1/2}\rangle: \quad \frac{3}{4}V_{\bar{N}N}(^1S_0) + \frac{1}{4}V_{\bar{N}N}(^3S_1),$$

$$|P\rangle = |P'\rangle = |^4S_{3/2}\rangle: \quad V_{\bar{N}N}(^3S_1),$$

$$|P\rangle = |P'\rangle = |^2P_{1/2}\rangle:$$
$$F_1^2 \cdot \left[\frac{1}{12}V_{\bar{N}N}(^3P_0) + \frac{3}{4}V_{\bar{N}N}(^1P_1) + \frac{1}{6}V_{\bar{N}N}(^3P_1)\right],$$

$$|P\rangle = |P'\rangle = |^4P_{1/2}\rangle: \quad F_1^2 \cdot \left[\frac{2}{3}V_{\bar{N}N}(^3P_0) + \frac{1}{3}V_{\bar{N}N}(^3P_1)\right],$$

$$|P\rangle = |P'\rangle = |^2P_{3/2}\rangle:$$
$$F_1^2 \cdot \left[\frac{3}{4}V_{\bar{N}N}(^1P_1) + \frac{1}{24}V_{\bar{N}N}(^3P_1) + \frac{5}{24}V_{\bar{N}N}(^3P_2)\right],$$

$$|P\rangle = |P'\rangle = |^4P_{3/2}\rangle: \quad F_1^2 \cdot \left[\frac{5}{6}V_{\bar{N}N}(^3P_1) + \frac{1}{6}V_{\bar{N}N}(^3P_2)\right],$$

Table 1
The energy shifts ΔE and decay widths of the $1s$ and $2p$ antiproton–deuteron atomic states in the approximation of undistorted deuteron core. The minus sign of the energy shifts means that the strong interaction is repulsive. The units are eV and meV for $1s$ and $2p$ states, respectively

	Paris98		DR2		KW		Data	
	ΔE	Γ	ΔE	Γ	ΔE	Γ	ΔE	Γ
$^2S_{1/2}$	−2445	1781	−2673	2380	−2478	2450		
$^4SD_{3/2}$	−2680	2822	−2668	2390	−2503	2469		
$^2P_{1/2}$	−186	584	17	896	99	657		
$^4P_{1/2}$	265	402	47	846	101	785		
$^2P_{3/2}$	−128	515	14	897	98	643		
$^4PF_{3/2}$	282	477	21	887	97	648		
$^4PF_{5/2}$	244	814	21	877	101	660		
$\Delta \bar{E}_{1s}, \bar{\Gamma}_{1s}$	−2602	2475	−2670	2387	−2494	2463	−1050 ± 250 [1]	1100 ± 750 [1]
								2270 ± 260 [2]
$\Delta \bar{E}_{2p}, \bar{\Gamma}_{2p}$	124	602	22	883	99	668	−243 ± 26 [2]	489 ± 30 [2]

$|P\rangle = |P'\rangle = |^4P_{5/2}\rangle$: $\quad F_1^2 \cdot V_{\bar{N}N}(^3P_2)$,

$|P\rangle = |P'\rangle = |^4D_{3/2}\rangle$: $\quad F_3^2 \cdot \left[\frac{1}{2} V_{\bar{N}N}(^3D_1) + \frac{1}{2} V_{\bar{N}N}(^3D_2) \right]$,

$|P\rangle = |P'\rangle = |^2F_{3/2}\rangle$: $\quad F_2^2 \cdot V_{\bar{N}N}(^3F_2)$,

$|P\rangle = |P'\rangle = |^4F_{5/2}\rangle$: $\quad F_2^2 \cdot \left[\frac{4}{9} V_{\bar{N}N}(^3F_2) + \frac{5}{9} V_{\bar{N}N}(^3F_3) \right]$,

$|P\rangle = |^4P_{3/2}\rangle, |P'\rangle = |^4F_{3/2}\rangle$: $\quad F_1 F_2 \cdot \frac{1}{\sqrt{6}} V_{\bar{N}N}(^3PF_2)$,

$|P\rangle = |^4P_{5/2}\rangle, |P'\rangle = |^4F_{5/2}\rangle$: $\quad F_1 F_2 \cdot \frac{2}{3} V_{\bar{N}N}(^3PF_2)$,

$|P\rangle = |^4S_{3/2}\rangle, |P'\rangle = |^4D_{3/2}\rangle$:

$$F_3 \cdot \left[\frac{1}{\sqrt{2}} V_{\bar{N}N}(^3SD_1) + \frac{1}{\sqrt{2}} V_{\bar{N}N}(^3SD_2) \right] \quad (16)$$

where $|P\rangle \equiv |JMLS\rangle$ and $|P'\rangle \equiv |JML'S\rangle$ are the $\bar{p}D$ atomic states. Both the $\bar{p}D$ and $\bar{N}N$ states in Eq. (16) are labelled as $^{2S+1}L_J$ with S, L and J being respectively the total spin, total orbital angular momentum and total angular momentum. The potentials $V_{\bar{N}N}$, being functions of $r_{13} = \sqrt{\lambda^2 + \rho^2/4 - \rho\lambda x}$, stand for the $\bar{N}N$ interactions for various $\bar{N}N$ states as indicated in the brackets.

The F_1, F_2 and F_3 in Eq. (16) are functions of only λ and ρ, taking the forms

$$F_1 = \begin{cases} 1 - \frac{1}{12} \frac{\rho^2}{\lambda^2}, & \rho < 2\lambda, \\ \frac{4\lambda}{3\rho}, & \rho > 2\lambda, \end{cases} \quad (17)$$

$$F_2 = \begin{cases} (1 - \frac{\rho^2}{4\lambda^2})^2, & \rho < 2\lambda, \\ 0, & \rho > 2\lambda, \end{cases} \quad (18)$$

$$F_3 = \begin{cases} {}_2F^1(1, -\frac{3}{2}, \frac{3}{2}, \frac{\rho^2}{4\lambda^2}), & \rho < 2\lambda, \\ \frac{5}{8} - \frac{3\rho^2}{32\lambda^2} + \text{Artanh}(\frac{2\lambda}{\rho})[\frac{3\lambda}{4\rho} - \frac{3\rho}{8\lambda} + \frac{3\rho^3}{64\lambda^3}], & \rho > 2\lambda, \end{cases}$$
$$\quad (19)$$

where ${}_2F^1(\alpha, \beta, \gamma, x)$ is the hypergeometric function and Artanh(x) the inverses hyperbolic tangent function.

3. Energy shifts and decay widths of $\bar{p}D$ atoms

It is not a simple problem to accurately evaluate the energy shifts and decay widths, especially wave functions of exotic atoms like protonium, pionium and antiproton–deuteron atoms, which are mainly bound by the Coulomb force, but also effected by the short range strong interaction. In this work we study the $\bar{p}D$ atoms in the Sturmian function approach which has been successfully applied to our previous works [14,15]. Employed for the $\bar{N}N$ interactions are various realistic $\bar{N}N$ potentials, namely, the Paris $\bar{N}N$ potentials of the 1994 version (Paris84), 1998 version (Paris98) and 2004 version (Paris04), the Dover–Richard $\bar{N}N$ potentials I (DR1) and II (DR2), and the Kohno–Weise $\bar{N}N$ potential (KW). In this preliminary work, we just limit our study to the approximation of undistorted deuteron core. However, one may see that the main conclusions of the work are free of this approximation.

Shown in Table 1 are the energy shifts and decay widths, which stem from the Paris98, DR2 and KW $\bar{N}N$ interactions, in the approximation of undistorted deuteron core. The theoretical results for other interactions like Paris84, Paris04 and DR1 are quite similar to the ones listed in Table 1. The wave function of the undistorted deuteron core is evaluated in the Bonn OBEPQ potential [16]. It is found that the theoretical results for the $1s$ $\bar{p}D$ atomic states are more or less the same by all the employed $\bar{N}N$ potentials. The predicted energy shifts are roughly as twice large as the experimental data. However, one may expect that the predictions of the potentials in question could be improved to some extent by solving the $\bar{p}D$ dynamical equation in Eq. (7) without any approximation. A better treatment of the deuteron core will yield lower $1s$ $\bar{p}D$ atomic states, hence smaller energy shifts. The theoretical results for the decay widths of the $1s$ $\bar{p}D$ atoms are also larger than the experimental data though not as far from the data as for the energy shifts. The predictions for the decay widths are also expected to be improved by treating the deuteron core more properly.

The theoretical predictions for the energy shifts of the $2p$ $\bar{p}D$ atomic states are totally out of line for all the $\bar{N}N$ potentials employed. The experimental data show that the averaged energy level of the $2p$ $\bar{p}D$ atoms is pushed up by the strong

interaction, the same as for the $1s$ $\bar{p}D$ atoms, but the theoretical results uniquely show the averaged energy level shifting down. It is unlikely to improve, by treating the deuteron core more accurately, the theoretical predictions of the $\bar{N}N$ potentials in question for the $2p$ $\bar{p}D$ energy shifts since a more accurate treatment of the deuteron core will lead to deeper $2p$ $\bar{p}D$ atomic states.

All the $\bar{N}N$ potentials employed in the work reproduce $\bar{N}N$ scattering data reasonably, but badly fail to reproduce the energy shifts of the $2p$ $\bar{p}D$ atoms. The investigation of the $\bar{p}D$ atoms may provide a good platform for refining the $\bar{N}N$ interaction, especially at zero energy since the energy shifts of the $2p$ $\bar{p}D$ atomic states are very sensitive to the $\bar{N}N$ strong interactions.

The research here is just a preliminary work, where a frozen, S-state deuteron is employed. The work may be improved at two steps, considering that the numerical evaluation is time-consuming. One may, at the first step, solve the $\bar{p}D$ dynamical equation in Eq. (7) by expanding the $\bar{p}D$ wave function in a bi-wave basis of the Sturmian functions, where a realistic nucleon–nucleon potential is employed but the deuteron core is assumed to be at the S-state. Such an evaluation is still manageable at a personnel computer but it may take a week or longer. We may compare the results of the improved work with the results here to figure out how important an unfrozen deuteron core is.

One may also consider, at the second step, to solve the $\bar{p}D$ dynamical equation in Eq. (7) by expanding the $\bar{p}D$ wave function in a bi-wave basis of the Sturmian functions without any approximation, where realistic nucleon–nucleon and nucleon–antinucleon potentials are employed and the deuteron core is allowed to be at both the S- and D-waves. It is certain that the numerical calculation will take longer time but, anyway, we will do it after we complete the first-step improvement.

Acknowledgements

References

[1] M. Augsburger, et al., Phys. Lett. B 461 (1999) 417.
[2] D. Gotta, et al., Nucl. Phys. A 660 (1999) 283.
[3] S. Wycech, A.M. Green, J.A. Niskanen, Phys. Lett. B 152 (1985) 308.
[4] G.P. Latta, P.C. Tandy, Phys. Rev. C 42 (1990) R1207.
[5] G.Q. Liu, J.-M. Richard, S. Wycech, Phys. Lett. B 2602 (1991) 15.
[6] V.A. Karmanov, K.V. Protasov, A.Yu. Voronin, Eur. Phys. J. A 8 (2000) 429.
[7] M. Pignone, M. Lacombe, B. Loiseau, R. Vinh Mau, Phys. Rev. C 50 (1994) 2710.
[8] B. El-Bennich, M. Lacombe, B. Loiseau, R. Vinh Mau, Phys. Rev. C 59 (1998) 2313.
[9] S. Wycech, B. Loiseau, AIP Conf. Proc. 796 (2005) 131.
[10] C.B. Dover, J.M. Richard, Phys. Rev. C 21 (1980) 1466.
[11] J.M. Richard, M.E. Sainio, Phys. Lett. B 110 (1982) 349.
[12] M. Kohno, W. Weise, Nucl. Phys. A 454 (1986) 429.
[13] M. Rotenberg, Adv. At. Mol. Phys. 6 (1970) 233.
[14] Y. Yan, R. Tegen, T. Gutsche, A. Faessler, Phys. Rev. C 56 (1997) 1596.
[15] P. Suebka, Y. Yan, Phys. Rev. C 70 (2004) 034006.
[16] R. Machleidt, K. Holinde, Ch. Elster, Phys. Rep. 149 (1987) 1.