



## รายงานการวิจัย

# การศึกษาเวฟเลตฟิลเตอร์อิงโดยใช้อินโคฮีเร้นท์ออฟติคอลเทคนิค Study of Wavelet Filtering Using Incoherent Optical Technique

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ได้รับทุนอุดหนุนการวิจัยจาก  
มหาวิทยาลัยเทคโนโลยีสุรนารี

ผลงานวิจัยเป็นความรับผิดชอบของหัวหน้าโครงการวิจัยแต่เพียงผู้เดียว



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Study of Wavelet Filtering Using Incoherent Optical Technique

ผู้วิจัย

หัวหน้าโครงการ

Asst. Prof. Dr. Joewono Widjaja

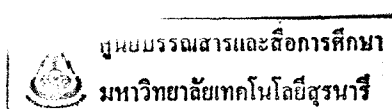
สาขาวิชาเทคโนโลยีเลเซอร์และโฟตอนิกส์

สำนักวิชาวิทยาศาสตร์

ได้รับทุนอุดหนุนการวิจัยจากมหาวิทยาลัยเทคโนโลยีสุรนารี ปีงบประมาณ 2542

ผลงานวิจัยเป็นความรับผิดชอบของหัวหน้าโครงการวิจัยแต่เพียงผู้เดียว

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## บทคัดย่อ

การศึกษาวิธีการใหม่สำหรับสร้างตัวกรองเวฟเล็ทด้วยวิธีการทางแสงแบบไม่โคเฮียเร้นท์ ซึ่งวิธีการแรกจะคำนวณตัวกรองเวฟเล็ทแบบหลายช่องทางโดยใช้การซ้อนกันเป็นแถวของตัวแปลงเวฟเล็ททางแสงแบบสองมิติ ซึ่งตัวแปลงทางแสงนี้จะเป็นการเชื่อมต่อกันระหว่าง holographic lens array (HLA) กับ optically addressed spatial light modulator (OASLM) ซึ่งกลุ่มของเวฟเล็ทจะถูกสร้างขึ้นมาเป็นแบบไม่โคเฮียเร้นท์จากหน้าจอโทรทัศน์ที่เชื่อมต่ออยู่กับเครื่องคอมพิวเตอร์ สำหรับวิธีการที่สองเราใช้ข้อดีจากคุณสมบัติของฟูเรียร์ทัศนศาสตร์ นั่นก็คือขนาดของการแปลงฟูเรียร์จะถูกเปลี่ยนไปตามความยาวคลื่นของแสงที่ใช้ ซึ่งการใช้แสงสีขาวที่มีการตอบสนองทางความถี่กว้าง จะทำให้กลุ่มของเวฟเล็ทถูกสร้างขึ้นมาจากเวฟเล็ทแม่ในทันทีทันใด ผลที่ได้คือตัวกรองเวฟเล็ทแบบหลายช่องทางสามารถสร้างได้จากตัวแปลงเวฟเล็ททางแสงเพียงชุดเดียว การเปรียบเทียบผลการทดลองเบื้องต้นและประสิทธิภาพของทั้งสองวิธีพบว่าการใช้วิธีที่สองจะได้จำนวนของช่องทาง และค่าผลคูณระหว่าง space กับ bandwidth (SBWP) สูงกว่า และจากการที่ไม่ต้องใช้ HLA และ OASLM ทำให้วิธีที่สองมีราคาถูกลงและง่ายกว่าวิธีแรก

## Abstract

New methods for implementing wavelet filtering by using incoherent optical technique are proposed. The first method computes a multi-channel wavelet filtering by using an array of the 2-D optical wavelet transformers. The optical transformers are constructed from a combination of a holographic lens array (HLA) and an optically addressed spatial light modulator (OASLM) in which a set of the daughter wavelets is incoherently generated from a TV monitor connected to a personal computer. In the second method, we take an advantage of the Fourier optics property where the Fourier transformation is inherently scaled by a wavelength of the illuminating light. By using a white light source having a wide spectral response, a set of daughter wavelets could be simultaneously generated from the mother wavelet. As the result the multi-channel wavelet filtering could be performed by a single 2-D optical WT. Preliminary experimental results and the throughput performance of the two methods are presented. In comparison with the first method, the second one has a higher number of channel and a space bandwidth product (SBWP). It is also low cost and simpler because it does not use the HLA and OASLM.

# Contents

<b>Acknowledgements</b> .....	i
<b>Thai Abstract</b> .....	ii
<b>English Abstract</b> .....	iii
<b>Contents</b> .....	iv
<b>List of Figures</b> .....	v
<b>Chapter 1 Introduction</b> .....	1
1.1 Wavelet transform .....	1
1.2 Literature review .....	2
1.3 Objective .....	3
<b>Chapter 2 Multi-Resolution Wavelet Filtering</b> .....	5
2.1 Methodology: Use of incoherent-coherent spatial light modulator .....	5
2.2 Methodology: Use of white light source .....	5
2.2.1 Multiple imaging using white light source .....	5
2.2.2 Multi-channel 2-D WT .....	7
2.2.3 Experimental results .....	8
2.2.4 Discussions .....	9
2.2.5 Conclusions .....	11
<b>Chapter 3 Conclusions</b> .....	12
<b>References</b> .....	13
<b>Appendix A</b> .....	15
<b>Curriculum Vitae</b> .....	20

# List of Figures

1.1 Block diagrams of multi-channel 2-D wavelet filtering (a) using incoherently generated wavelet filter and (b) using incoherent light source .....	3
2.1 Optical 2-D WT by using a 2-D grating and a white light source .....	5
2.2 (a) Multiple images of the star target (b) resultant multi-channel wavelet filtering .....	8
2.3 Polygonal generation of optical multi-channel 2-D wavelet filtering .....	10

# Chapter 1

## Introduction

### 1.1 Wavelet transform

A wavelet transform (WT) has already been known to be useful for analyzing non-stationary signals [1]. The WT decomposes a signal into a set of elementary functions which are derived from a unique function called the mother wavelet by means of dilations and translations. Unlike a short time Fourier transform [2], the WT is a multi-resolution signal representation that gives a better time resolution for high frequency than for low frequency components. This is consistent with a nature of the non-stationary signal whose high frequency component lasts in a relatively short time. This unique property of the WT has been successfully used to extract feature of images by Mallat *et al.* [3].

The WT of a two-dimensional (2-D) signal pattern  $f(x,y)$  is defined by [1]

$$W_f(a_x, a_y, b_x, b_y) = \int_{-\infty}^{+\infty} \int f(x, y) h_{a_x, a_y, b_x, b_y}^*(x, y) dx dy, \quad (1.1)$$

where  $h_{a_x, a_y, b_x, b_y}(x, y)$  is the daughter wavelet generated by translating and dilating the mother wavelet  $h(x, y)$  as

$$h_{a_x, a_y, b_x, b_y}(x, y) = \frac{1}{\sqrt{a_x a_y}} h\left(\frac{x - b_x}{a_x}, \frac{y - b_y}{a_y}\right). \quad (1.2)$$

In Fourier domain, Eq. (1) can be written as

$$W_f(a_x, a_y, b_x, b_y) = \sqrt{a_x a_y} \int_{-\infty}^{+\infty} \int F(u, v) H^*(a_x u, a_y v) \times \exp\{i2\pi(b_x u + b_y v)\} du dv, \quad (1.3)$$

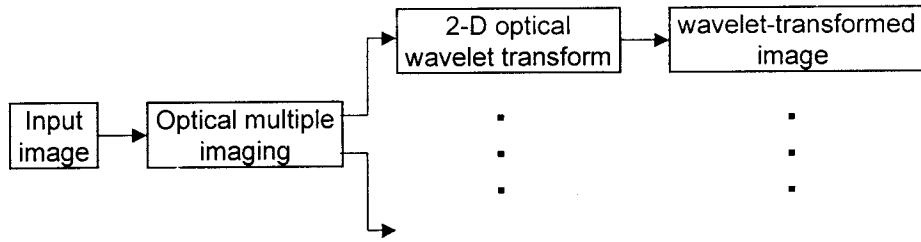


where  $F(u, v)$  and  $H(a_x u, a_y v)$  are the Fourier transform of the signal  $f(x, y)$  and the daughter wavelet  $h_{a_x a_y b_x b_y}(x, y)$ , respectively.

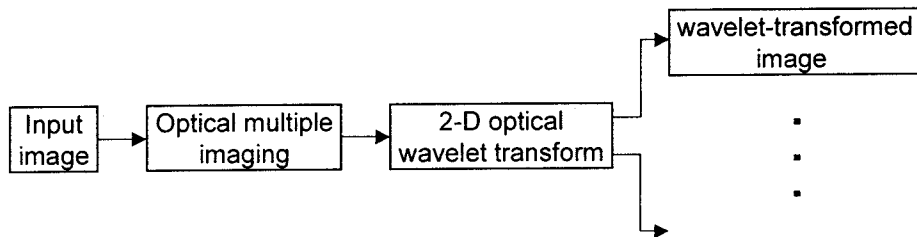
## 1.2 Literature review

In the field of optics, for a given pair of dilation parameter  $a_x$  and  $a_y$ , Eqs. (1) and (3) can be processed in the spatial and in the Fourier domains, respectively. The spatial-domain processing could be accomplished by using a joint-transform correlator (JTC) where the analyzing wavelet  $h_{a_x a_y b_x b_y}(x, y)$  was encoded as a phase pattern [4], while in the Fourier-domain processing  $H(a_x u, a_y v)$  could be synthesized as either real or holographic filters [5-7]. In this approach, the multi-resolution image representation by using the WT was then accomplished by changing successively the synthesized wavelets with different dilations. However, a time delay introduced by this operation reduces significantly computation speed of the WT. In order to take full advantage of parallel nature of optical computer, spatial-multiplexed 2-D WT by using multi-reference matched filters [8-10] and multi-input JTC [11] have also been suggested. Although these proposed methods could generate simultaneously several WT of image at given resolutions, they require inherently two-step processes that are synthesis and correlation for their computations. Therefore, there is no flexibility to modify dilations of the analyzing wavelet once it was synthesized. Moreover, the use of the multi-input JTC is hard to be practically applied to multi-resolution image analysis because the loss of light to useless outputs and the cross talk effect increases severely when the number of the channels increases. Recently, the 2-D optical implementation WT using a white light source has been proposed by Zalevsky [12], where according to Fourier optics the spectrum size of the input image is inversely proportional to the wavelength of the light source. Since the white light source contains wide band of optical spectrum, several different dilations could be simultaneously obtained. However, this method has a drawback that in

order to obtain the transformed output, an appropriate color filter must be sequentially placed at the output plane.



(a)



(b)

Fig. 1.1. Block diagrams of multi-channel 2-D wavelet filtering (a) using incoherently generated wavelet filter and (b) using incoherent light source

### 1.3 Objective

In order to eliminate these problems, we investigate two novel optical methods for implementing multi-channel wavelet filtering. Figure 1 shows two block diagrams of the proposed method. In both methods, we first generate multiple images of the input image to be analyzed by using a coherent and white light sources, respectively. The generation of multiple images is necessary in order to perform simultaneously multi-channel WT. In the first method shown in Fig. 1(a), the multi-channel WT are performed by an array of the 2-D optical wavelet transformers consisting of a combination of the HLA and the OASLM. In each channel of the 2-D transformer, a different daughter wavelet is employed. We generate

incoherently a set of the daughter wavelets by using a TV monitor connected to a personal computer.

In the second method, the white light source is employed for first providing wavelength multiplexing. The optical Fourier transformation optics property inherently scaled by a wavelength of the illuminating light is used to generate simultaneously a set of daughter wavelets. The white light source is also used for generating multiple images of the input pattern. In comparison with the first method, the position of the generated multiple images are determined by both the grating spacing and the spectral response of illuminating light. Therefore, the number of the generated multiple images is higher than the first method. Furthermore by employing a single 2-D optical WT, the multi-resolution wavelet filtering could be performed in parallel. The desired wavelet filtering could be simultaneously obtained, provided a set of appropriate color filters is simultaneously placed at the output plane.

# Chapter 2

## Multi-Resolution Wavelet Filtering

### 2.1 Methodology: Use of incoherent-coherent spatial light modulator

In this method, the multi-channel WT are performed by an array of the 2-D optical wavelet transformers consisting of a combination of the HLA and the OASLM. Each channel of the 2-D transformer employs a different daughter wavelet (See Appendix A).

### 2.2 Methodology: Use of white light source

A schematic diagram of the second proposed method consisting of two cascaded 4-f optical setups is illustrated in Fig. 2.1. The first setup similar to the setup discussed in the Appendix A performs the multiple imaging, while the second setup implements the 2-D WT. In the second proposed method, instead of the array of the daughter wavelets a single mother wavelet  $H(u,v)$  is employed. A set of daughter wavelets is automatically generated by using the wavelength of the illuminating light source.

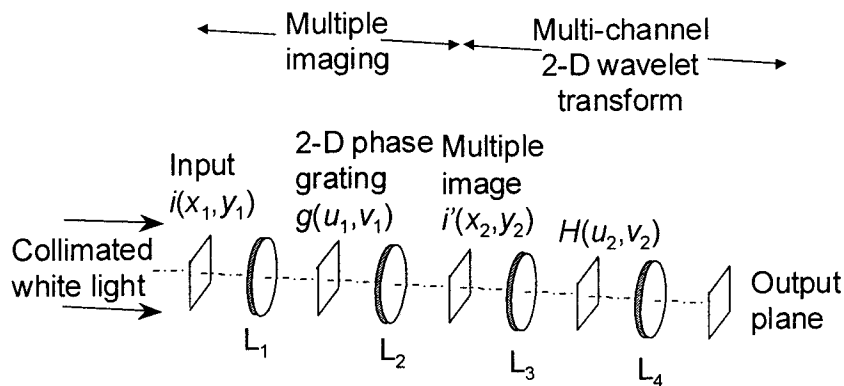


Fig. 2.1. Optical 2-D WT by using a 2-D grating and a white light source.

#### 2.2.1 Multiple imaging using white light source

Under the white light illumination, the optical field appears in the Fourier plane of the lens  $L_1$  can be written as

$$\begin{aligned}
I(u_1, v_1, \lambda) &= S(\lambda) \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} i(x_1, y_1) \exp[2\pi i(u_1 x_1 + v_1 y_1)] dx_1 dy_1 \\
&= S(\lambda) I(u_1, v_1) = S(\lambda) I\left(\frac{x_F}{\lambda f}, \frac{y_F}{\lambda f}\right),
\end{aligned} \tag{2.1}$$

where  $S(\lambda)$  and  $i(x_1, y_1)$  stand for the spectral response of the white light and the input pattern, respectively.  $f$  is the focal length of the lens, while  $u_1$  and  $v_1$  are the spatial-frequency coordinates in the horizontal and vertical directions, respectively. They are related to the actual coordinates  $(x_F, y_F)$  by  $x_F = \lambda f u_1$  and  $y_F = \lambda f v_1$ . After passing the 2-D phase grating [13], the optical field becomes

$$O(u_1, v_1, \lambda) = I(u_1, v_1, \lambda) \sum_{m=1}^N \sum_{n=1}^N \delta(u_1 - m\Delta u_1) \delta(v_1 - n\Delta v_1), \tag{2.2}$$

where the summation of two delta functions correspond to the grating function.  $\Delta u_1$  and  $\Delta v_1$  stand for the spatial frequencies of the grating in the horizontal and the vertical directions, respectively. The Fourier transformation done by the lens  $L_2$  generates multiple images of the input pattern at the back focal plane of the lens  $L_2$  given by

$$\begin{aligned}
i'(x_2, y_2, \lambda) &= i(x_2, y_2, \lambda) \otimes \sum_{m=1}^N \sum_{n=1}^N \delta(x_2 - m\Delta x_2) \delta(y_2 - n\Delta y_2) \\
&= \sum_{m=1}^N \sum_{n=1}^N i(x_2 - m\Delta x_2, y_2 - n\Delta y_2, \lambda),
\end{aligned} \tag{2.3}$$

where  $\otimes$  denotes the convolution operator.  $\Delta x_2$  and  $\Delta y_2$  correspond to the distances between centers of the generated scenes in the horizontal and the vertical directions, respectively.

Their relationship with the spatial frequencies of the grating is given by

$$\Delta x_2 = \frac{1}{\Delta u_1} = \frac{\lambda f}{\xi_x} \quad \text{and} \quad \Delta y_2 = \frac{1}{\Delta v_1} = \frac{\lambda f}{\xi_y}, \tag{2.4}$$

with  $\xi_x$  and  $\xi_y$  are the actual grating spacing in the horizontal and vertical directions, respectively. Equation (2.4) shows that the spatial distance between multiple images  $\Delta x_2$  and  $\Delta y_2$  are determined by the wavelength of the illuminating light. Since this optical system uses white light source, therefore the generated optical field  $i'(x_2, y_2, \lambda)$  consists of multiple images from different wavelengths that may overlap each other.

### 2.2.2 Multi-channel 2-D WT

By considering the optical field  $i'(x_2, y_2, \lambda)$  as the input of the second 4-f optical setup, at the back Fourier plane of the lens  $L_3$ , we obtain

$$\begin{aligned} I'(u_2, v_2, \lambda) &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} i'(x_2, y_2, \lambda) \exp[2\pi i(u_2 x_2 + v_2 y_2)] dx_2 dy_2 \\ &= I(u_2, v_2, \lambda) \sum_{m=1}^N \sum_{n=1}^N \delta\left(u_2 - \frac{m}{\Delta x_2}\right) \delta\left(v_2 - \frac{m}{\Delta y_2}\right). \end{aligned} \quad (2.5)$$

In order to perform the multi-channel WT, a binary transparency of the Fourier-transformed mother wavelet  $H(u, v)$  is placed at the back focal plane of the lens  $L_3$ . The optical field distribution of the product  $I(u_2, v_2, \lambda)H(u_2, v_2)$  is then Fourier transformed by the lens  $L_4$ . The result given by

$$\begin{aligned} o(b_x, b_y, \lambda) &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} I'(u_2, v_2, \lambda) H(u_2, v_2) \exp[2\pi i(b_x u_2 + b_y v_2)] du_2 dv_2 \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} I(u_2, v_2, \lambda) \sum_{m=1}^N \sum_{n=1}^N \delta\left(u_2 - \frac{m}{\Delta x_2}\right) \delta\left(v_2 - \frac{m}{\Delta y_2}\right) H(u_2, v_2) \\ &\quad \times \exp[2\pi i(b_x u_2 + b_y v_2)] du_2 dv_2 \end{aligned} \quad (2.6)$$

could be obtained at the output plane. By using the shift theorem of the Fourier transform, Eq. (2.6) can be simply reduced to

$$o(b_x, b_y, \lambda) = S(\lambda) \sum_{m=1}^N \sum_{n=1}^N \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} i(x + m\Delta x_2, y + m\Delta y_2) h\left(\frac{b_x - x}{\lambda f}, \frac{b_y - y}{\lambda f}\right) dx dy, \quad (2.7)$$

film was then installed at the back focal plane of the lens  $L_2$ , while the 1-D 80 lines/mm vertical grating was placed midway between lenses of the first 4-f optical setup. Finally, a digital camera Sony MVC-FD71 was used to capture the resultant wavelet-transformed images.

Figures 2.2 (a) and (b) show the multiple images of the target generated at the back focal plane of the lens  $L_2$  and the multi-channel WT obtained at the output plane, respectively. In Fig. 2.2 (a), two sets of images of the target corresponding to the 543nm and 632.8nm wavelengths are spatially superimposed due to a diffractive nature of the grating. In each set of the output, three images associating with the  $-1$ , the  $0$ , and the  $+1$  order of diffraction patterns emerge. Since the spatial separation  $\Delta x_2$  of Eq. (2.4) is linearly proportional to the wavelength, the longer wavelength gives a wider spatial separation compared than the shorter one. Furthermore, the overlapping areas of the images appear yellow because the superposition of the red and green light sources. Figure 2.2 (b) shows that edge features of the wavelet-transformed outputs are enhanced in accordance with the wavelength of the illuminating light source. The scaling with the wavelength of 543nm gives a stronger edge enhancement effect in comparison with the 632.8nm. This result confirms the feasibility of our proposed method because the daughter wavelet having a small scaling factor works as a bandpass filter with high center spatial frequency, while a large scaling factor provides lower spatial frequency response. Therefore, the effect of the scaling with the shorter wavelength is to enhance strongly edge features of the input pattern. In the summary, optical implementation of three-channel WTs has been demonstrated by using multiplexing of two coherent light sources with different wavelength.

#### **2.2.4 Discussions**

A throughput performance of the proposed method could be analyzed by using the number of

channels which is determined by the number of gratings. By assuming that each grating produces three diffraction patterns, the number of channel  $N$  could be expressed as

$$N = 2n + 1, \quad (2.8)$$

where  $n$  is the number of gratings. Figure 2.3 illustrates an example of a polygonal construction of nine channel WT outputs generated by using four crossed 1-D gratings having an angular separation of  $45^\circ$ . Each WT output is illustrated as a circle with a diameter of  $d$ .

For  $n$  gratings, the angular separation is given by

$$\theta = \pi/n. \quad (2.9)$$

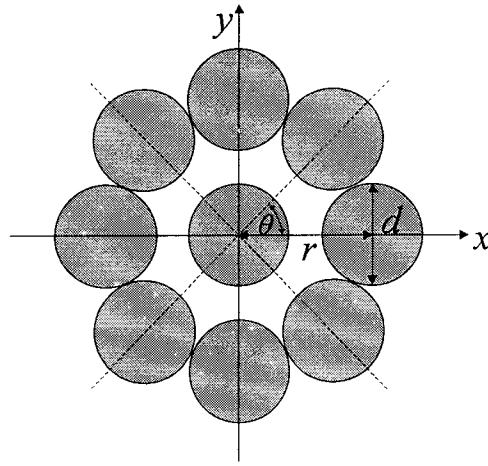


Fig. 2.3. Polygonal generation of optical multi-channel 2-D wavelet filtering.

In order to avoid cross-talk between output channels, the output image corresponding to the shortest visible wavelength  $\lambda_0$  must appear at a radial position of

$$r = \frac{\lambda_0 f}{\xi}, \quad (2.10)$$

where  $\xi$  stands for the grating spacing of all gratings. From Fig. 2.3, it is clear that for any number of channels the optical system must have

$$r = \frac{d}{2 \sin(\pi/2n)}. \quad (2.11)$$



Substitution of Eqs. (2.8) and (2.10) into Eq. (2.11) yields

$$\xi = \frac{2\lambda_0 f \sin(\pi/N - 1)}{d}. \quad (2.12)$$

Equation (2.12) describes the maximum number of channels that can be accomplished once the input object size  $d$  and the grating spacing  $\xi$  have been fixed in a particular optical system.

### **2.2.5 Conclusions**

We have proposed and demonstrated experimentally the multi-resolution 2-D optical WT by using a combination of the grating and the polychromatic light source. The experimental results verify the feasibility of our proposed method. Besides the width of the spectral response of the illuminating light, the number of channels is inversely determined by the grating spacing.

## Chapter 3

### Conclusions

We have proposed two systems for implementing 2-D wavelet filtering by using incoherent optical methods. In our proposed methods, multiple images of the input image to be analyzed are firstly generated by using either a coherent or white light sources. The generation of multiple images is necessary in order to perform simultaneously multi-channel WT.

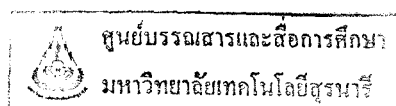
In the first method, the multi-channel WT is performed in parallel by an array of the 2-D optical wavelet transformers consisting of a combination of the HLA and OASLM. In each channel of the 2-D wavelet transformer, a different daughter wavelet is employed. We generate incoherently a set of the daughter wavelets by using a TV monitor connected to a personal computer.

In the second method, the white light source is employed for providing both wavelength multiplexing and generating multiple images of the input pattern. Since the optical Fourier transformation optics property inherently scaled by a wavelength of the illuminating light, a set of daughter wavelets could be simultaneously generated. In comparison with the first method, the position of the generated multiple images are not only determined by the grating spacing but also by the spectral response of the illuminating light. As the result, the number of processing channels is higher than the first method.

We have also analyzed the throughput performance of the system. The result shows that the second method has a higher degree of SBWP than the first [13] because the use of the HLA could be avoided. Furthermore, our second proposed method is low cost and simpler compared than the first because the use of HLA and OASLM could be avoided.

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# Appendix A

## Dynamic Optical Processing of 2-D Wavelet Transforms

Joewono Widjaja

Institute of Science, Suranaree University of Technology, Nakorn Ratchasima 30000, Thailand

A new optical method for implementing multichannel two-dimensional wavelet transform is proposed. The method relies on a combination of a holographic lens array and optically-addressed spatial light modulator. Limitations of the method are discussed.

**Key words:** Wavelet transforms, information processing, Fourier optics.

### 1. INTRODUCTION

A wavelet transform (WT) is already known to be useful for analyzing nonstationary signals [1]. The WT decomposes a signal into a set of elementary functions which are derived from a unique function called the mother wavelet by means of dilations and translations. Unlike a short time Fourier transform [2], the WT is a multiresolution signal representation which gives a better time resolution for high frequency than for low frequency components. This is consistent with the nature of the nonstationary signal whose high frequency component lasts a relatively short time. This unique property of the WT has been successfully used to extract the feature of images by Mallat et al. [3].

The WT of a two-dimensional (2-D) signal pattern  $f(x, y)$  is defined by [1]

$$W_f(a_x, a_y, b_x, b_y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) h_{a_x, a_y, b_x, b_y}^*(x, y) dx dy, \quad (1)$$

where  $H_{a_x, a_y, b_x, b_y}(x, y)$  is the daughter wavelet generated by translating and dilating the mother wavelet  $h(x, y)$  as

$$h_{a_x, a_y, b_x, b_y}(x, y) = \frac{1}{\sqrt{a_x a_y}} h\left(\frac{x - b_x}{a_x}, \frac{y - b_y}{a_y}\right). \quad (2)$$

In the Fourier domain, Eq. (1) can be written as

$$W_f(a_x, a_y, b_x, b_y) = \sqrt{a_x a_y} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(u, v) H^*(a_x u, a_y v) \times \exp(i2\pi(b_x u + b_y v)) du dv, \quad (3)$$

where  $F(u, v)$  and  $H(a_x u, a_y v)$  are the Fourier transform of the signal  $f(x, y)$  and the daughter wavelet  $H_{a_x, a_y, b_x, b_y}(x, y)$ , respectively. In field optics, for a given pair of dilation parameters  $a_x$  and  $a_y$ , Eqs. (1) and (3) can be processed in the spatial and Fourier domains, respectively. In the spatial-domain processing, the analyzing wavelet  $H_{a_x, a_y, b_x, b_y}(x, y)$  was encoded as a phase pattern [4], while  $H(a_x u, a_y v)$  was synthesized as either real or holographic-matched filters in the Fourier-domain processing [5-7]. In this approach, the multiresolution image representation using the WT was accomplished by changing successively the synthesized wavelets with different dilations. However, a time delay introduced by this operation reduces significantly computation speed of the WT. In order to take full advantage of the parallel nature of the optical computer, spatial-multiplexed 2-D WT's using multireference matched filters [8-10] and multi-input JTC [11] have also been suggested. Although these proposed method could generate simultaneously several WT-based image analysis at given resolutions, they require inherently two-step processes that are synthesis

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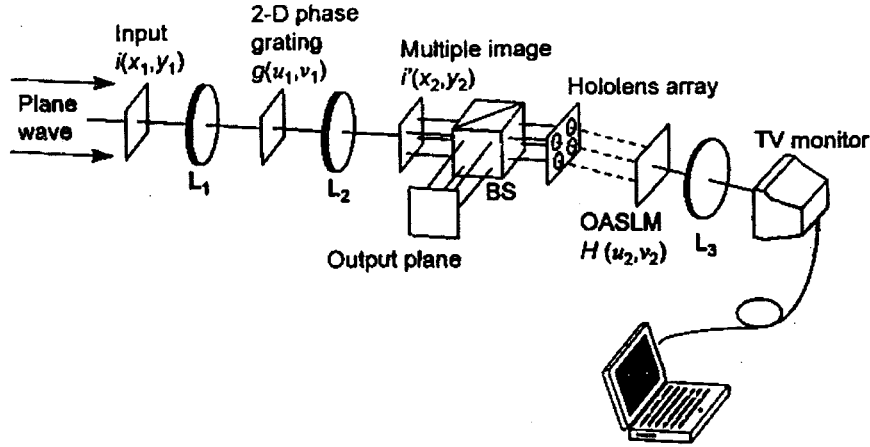


Fig. 1. Schematic diagram of the dynamic optical processing of 2-D wavelet transforms.

and correlation for their computations. Therefore, there is no flexibility to modify dilations of the analyzing wavelet once it was synthesized. Moreover, the use of the multi-input JTC is difficult to apply practically to multiresolution image analysis because the loss of light to useless outputs and the crosstalk effect increases severely when the number of the channels increases.

In order to obviate these problems, we propose a novel method for generating multichannel WT-based image analysis by using an optically-addressed spatial light modulator (OASLM) as the reconfigurable dynamic device for displaying the wavelet  $H(a, \mu, a, \nu)$ . Therefore, it is adaptive, since a reconfiguration of the wavelet is flexible and can be done in real time.

## 2. OPTICAL MULTICHANNEL 2-D WT

Figure 1 shows schematically a basic optical architecture for implementing dynamic multichannel WT-based image analysis. Here, a computation of four different resolutions of the input scene corresponding to four different scales of wavelets is illustrated as an example. The architecture consists of two setups. The first setup generates optically multiple images of the input scene by using a binary phase grating [12], while the second one performs the multichannel 2-D WT via a holographic lens array (ALA).

### 2.1. Multiple imaging

In order to compute multichannel WTs, an array of the input scenes are first generated using a 4-f optical setup. The input scene  $i(x_1, y_1)$  placed at a front focal plane of a Fourier transforming lens  $L_1$  is illuminated by a collimated coherent light. Its Fourier spectrum  $I(u_1, v_1)$  is filtered by a binary phase grating positioned at the back focal plane of the lens  $L_1$ . In order to abbreviate the tedious mathematical representation and yet model the sampling nature of the grating, the grating is defined by [12]

$$g(u_1, v_1) = \sum_{m=1}^N \sum_{n=1}^N \delta(u_1 - m\Delta u_1) \delta(v_1 - n\Delta v_1), \quad (4)$$

where  $\Delta u_1$  and  $\Delta v_1$  are the periods of the grating in the horizontal and the vertical directions, respectively. Therefore, the optical field immediately behind the grating is

$$O(u_1, v_1) = I(u_1, v_1) \sum_{m=1}^N \sum_{n=1}^N \delta(u_1 - m\Delta u_1) \delta(v_1 - n\Delta v_1), \quad (5)$$

By Fourier transforming this optical field, the array of the scene is obtained at the back focal plane of the lens  $L_2$  as

$$\begin{aligned}
 i'(x_2, y_2) &= i(x_2, y_2) \otimes \sum_{m=1}^N \sum_{n=1}^N \delta(x_2 - m\Delta x_2) \delta(y_2 - n\Delta y_2) \\
 &= \sum_{m=1}^N \sum_{n=1}^N i(x_2 - m\Delta x_2, y_2 - n\Delta y_2),
 \end{aligned}
 \tag{6}$$

where  $\otimes$  denotes the convolution operator. Here,  $\Delta x_2$  and  $\Delta y_2$  corresponding to the distances between centers of the generated scenes in the horizontal and the vertical directions are inversely proportional to  $\Delta u_1$  and  $\Delta v_1$ , respectively.

### 2.2. Multi-channel 2-D WT

Simultaneous computation of the multichannel 2-D WT can be done by utilizing the HLA [13]. The HLA is fabricated in such a way that the separation between centers of lenses in the horizontal and the vertical directions equals to  $\Delta x_2$  and  $\Delta y_2$ , respectively. By considering the multiple image  $i'(x_2, y_2)$  is located at the front focal plane of the HLA, multiple Fourier spectra can be simultaneously obtained at the back focal plane of the HLA. The spectrum in each channel is given by

$$\begin{aligned}
 I_{mn}(u_2, v_2) &= I_{mn}(u_2 - m\Delta u_2, v_2 - n\Delta v_2); \\
 &\text{for } m, n = 1, \dots, N
 \end{aligned}
 \tag{7}$$

where  $\Delta u_2$  and  $\Delta v_2$  stand for the distances between center of two spectra in the horizontal and vertical directions, respectively.

In order to perform multichannel WT computations, Fourier-transformed wavelets  $H(a_x u, a_y v)$  for given dilations are computed and stored into the computer. They are simultaneously displayed through a TV monitor and imaged to the write side of the OASLM positioned at the back focal plane of the holographic lens array via a lens  $L_3$ . Here, it is assumed that  $H(a_x u, a_y v) = |H(a_x u, a_y v)| = |H(a_x u, a_y v)|^2$ , because the Fourier transformation of real and symmetric mother wavelet is a positive function [8]. For a given dilation, the Fourier-transformed daughter wavelet can be written as

$$\begin{aligned}
 H_{mn}(a_x u_2, a_y v_2) &= H_{mn}[a_x(u_2 - m\Delta u_2), a_y(v_2 - n\Delta v_2)]; \\
 &\text{for } m, n = 1, \dots, N.
 \end{aligned}
 \tag{8}$$

$H_{mn}(a_x u, a_y v)$  in each channel is then read out by the optical field  $T_{mn}(u_2, v_2)$ . The resultant optical field in each channel is

$$\begin{aligned}
 O'_{mn}(u_2, v_2) &= I'_{mn}(u_2, v_2) H_{mn}(a_x u_2, a_y v_2) \\
 &= I_{mn}(u_2 - m\Delta u_2, v_2 - n\Delta v_2) H_{mn}[a_x(u_2 - m\Delta u_2), a_y(v_2 - n\Delta v_2)]; \\
 &\text{for } m, n = 1, \dots, N
 \end{aligned}
 \tag{9}$$

On the way back from the read side of the OASLM, the optical field  $O'(u_2, v_2)$  is Fourier transformed by the HLA and picked up by a beam splitter BS. As a result, multiresolution 2-D WT  $W_{mn}(a_x, a_y, b_x, b_y)$  of the input scene appears simultaneously at the output plane. Note that the dynamic image analysis using the WT could be done by reconfiguring the displayed  $H_{mn}(a_x u, a_y v)$  in real time.

### 3. DISCUSSION

In our proposed method, maximization of the number of channels can be done by decreasing the size of the input image  $D$ , and the holographic lens  $D_H$ . From Fig. 2, a further size reduction may cause crosstalk between channels.

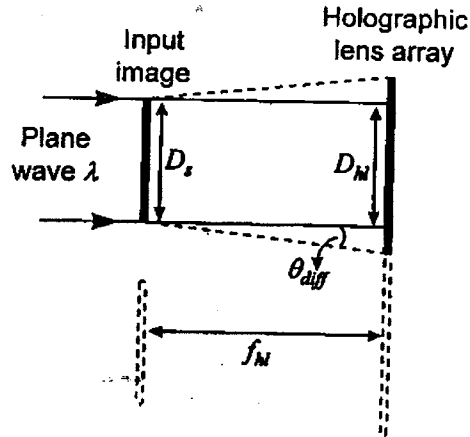


Fig. 2. Schematic diagram for analysis of the system limitations.

However, the cross talk at the lens plane may be avoided when the following expression is valid

$$D_{HL} = D_s + 2f_{HL} \tan \theta_{diff}, \tag{10}$$

where  $\theta_{diff} = \arcsin(\lambda/D_s)$  stands for the maximum diffraction angle.  $\lambda$  is the wavelength of the collimated coherent light, while  $f_{HL}$  is the focal length of the HLA. When  $\theta_{diff}$  is so small that  $2f_{HL} \tan \theta_{diff} \ll D_s$ , then

$$D_{HL} \cong D_s. \tag{11}$$

Furthermore, in order to have a clear guideline for realizing our proposed method, the concept of a space-bandwidth product (SBWP) is used. The SBWP of the input image  $f(x_1, y_1)$  is given by [14]

$$SBWP = \Delta u_s \Delta v_s D_s^2 = \Delta f_s^2 D_s^2, \tag{12}$$

where  $\Delta u_s$  and  $\Delta v_s$  are the maximum spatial frequencies of the input image in the horizontal and the vertical directions, respectively. For simplicity, it is assumed that  $|\Delta u_s| = |\Delta v_s| = |\Delta \xi_s|$ . In order to avoid overlapping spectra between each channel, the size of each spectrum at the Fourier-transform plane should be equal to the size of the holographic lens  $\lambda f_{HL} \Delta \xi_s \leq D_{HL}$  or

$$\Delta \xi_s \leq \frac{D_{HL}}{\lambda f_{HL}}. \tag{13}$$

Therefore, in terms of the parameters of the holographic lens, the SBWP of the input image is

$$SBWP = \frac{D_{HL}^2}{(\lambda F_H)^2}, \tag{14}$$

where  $F_H$  is the *F-number* of the holographic lens [14]. Equation (14) shows the dependency of the SBWP of the analyzed input image upon the parameters of the HLA, where use of high *F-number* HLA should be avoided since it reduces the SBWP of the analyzed input image. This equation also describes general criteria for designing the HLA.

#### 4. CONCLUSION

We have proposed a novel method for implementing optically multichannel image analysis based on the WT. We have also derived mathematically that the SBWP of the analyzed input image is dependent on the parameters of



the HLA. Our method relies on the HLA-OASLM combination. The use of the OASLM which may be addressed in real time by means of the TV monitor displays simultaneously multianalyzing wavelets for given resolutions, while the HLA is used for performing space-variant Fourier transformation. Therefore, in comparison with previously reported methods [4-11], our proposed method is suitable for adaptive image analysis.

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# CURRICULUM VITAE

**1. Name:** Joewono Widjaja

**2. Current position:** Assistant Professor

**3. Educational background:**

1986 Bachelor of Engineering (Electronic), Satya Wacana Univ., Indonesia

1991 Master of Engineering (Electronic), Hokkaido Univ., Japan

1994 Doctor of Engineering (Electronic), Hokkaido Univ., Japan

**4. Field of specialization:**

Optical information processing

**5. List of Publications:**

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