USE OF WAVELET TRANSFORM FOR SIZING PARTICLE

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ABSTRACT: A new method for sizing particles from in-line Fraunhofer holograms by using wavelet transform is proposed. The amplitude transmittance of the holograms is a modulation of a chirp signal with an envelope function whose minima is proportionally equal to the product of the particle size and the spatial frequency of the chirp function. By wavelet transforming the hologram and taking an absolute value of its resultant transformation, the spatial frequency at the minima positions can be obtained. The particle size which is merely a function of this frequency can then be calculated. Feasibility study of the proposed method is done by conducting simulation and experimental verifications for line object.

KEYWORDS: Fraunhofer holography, particle sizing, wavelet transform

1. INTRODUCTION

In-line particle holography is a useful method for measuring particle size and its spatial position. The in-line particle hologram is generated by recording an interference pattern of a transmitted and a diffracted waves from particles illuminated by coherent light. When a distance z between the particles and the recording plane satisfies the far-field condition, an amplitude transmittance of the in-line Fraunhofer hologram of a small opaque or semi-transparent line object with a radius of a can be mathematically expressed as (Tyler and Thompson 1976)

$$I(x) = 1 - \frac{4a}{\sqrt{\lambda z}}\cos\left(\frac{\pi x^2}{\lambda z} - \frac{\pi}{4}\right)\left[\frac{\sin\frac{2\pi a x}{\lambda z}}{\frac{2\pi a x}{\lambda z}}\right] + \frac{4a^2}{\lambda z}\left[\frac{\sin\frac{2\pi a x}{\lambda z}}{\frac{2\pi a x}{\lambda z}}\right]^2, \qquad (1)$$

where λ is the wavelength of the illuminating light. The second term of Eq. (1) predominates in the interference pattern; because the first term corresponding to the directly transmitted light is a constant background, while the amplitude of the square of a sinc function in the third term is much smaller than the other terms. In the second term, the recording distance z and the particle size a are encoded in the frequency of the chirp signal and the sinc function, respectively. In order to extract the desired information, the hologram is illuminated with the same coherent light. The information about the particle size can be obtained by analyzing the reconstructed image of the particle. However, in real applications, we may deal with a huge number of particles. As a consequence, the conventional reconstruction process is very tedious and time consuming. In order to obviate these problems particle sizing from in-line particle holograms by using absolute value of the wavelet transform (WT) is proposed.

2. METHOD

From the second term of Eq. (1), the frequency of the chirp signal equal to $x/\lambda z$, while the minima positions of the interference pattern corresponding to the zero-crossing positions of the sinc function appear at $2ax/\lambda z = n$, where n is integer determined by the order of minima. Thus, the frequencies of the chirp signal at the minimum positions are found to be $n/2a$. In our proposed method, a CCD sensor is employed to capture the hologram of the particle. After recording, the WT of the interference pattern can be digitally computed by using a computer. The reason of using the WT is that the frequency of the chirp is a function of position. Since the WT is an efficient mathematical tool for space-frequency analysis of the non-stationary signals, the desired frequency information at the minima position can be obtained by using the WT. Although another transforms such as a short time Fourier transform and a Wigner distribution function provide such analysis their time-frequency resolution are fixed. Therefore they are not suitable for analyzing non-stationary signals (Goswami and Chan 1999). In contrast, the inherent multi-resolution property of the WT allows us to decompose the analyzed signal in such a way that a good time and frequency resolution can be achieved at the high and low frequency portion of signal, respectively. It is found that by taking the absolute values of the resultant WT output, the frequencies of the chirp signal at the minimum positions can be obtained. The particle size can then be calculated because it is inverse proportional to these frequencies. In this work, all digital computations were conducted by using the Matlab 6.1.

Mathematically, the WT of a space signal $s(x)$ is defined as (Kronland-Marlinet, Morlet and Grossman 1987)

$$W(t,d)=\frac{1}{\sqrt{d}}\int_{-\infty}^{\infty}g^{*}\left(\frac{x-t}{d}\right)s(x)dx, \tag{2}$$

where d and t are the dilation and the translation parameters, respectively. In the computation process, the compactly supported daughter wavelet which is a translated and dilated version of its mother wavelet $g(x)$ is multiplied by the analyzed signal. The integration of the resultant multiplication determine the degree of similarity between the daughter wavelet and the signal in the portion that is subtended by the daughter wavelet. Thus, in the WT domain, a high correlation peak is generated at the translation t and dilation d when the frequency of the daughter wavelet matches to the frequency of the signal. It is found that the amplitude of the absolute value of this correlation peak is determined by the absolute value of the envelope function of the analyzed signal. Therefore, the information about the envelope function is preserved into the amplitude of the absolute value of the WT.

3. RESULTS AND DISCUSSION

The feasibility of our proposed method was verified by analyzing the hologram of an optical fiber having the diameter of 124.96 μm. These holograms were both simulated and optically generated under illumination of the coherent light operating at the wavelength of 543.5 nm. In the case of the optical generation, the hologram was recorded by using a CCD camera HAMAMATSU C5948 with the resolution of 640 × 480 pixels in the area of 8.3 × 5.3 mm. Figure 1 shows the simulated in-line hologram of the fiber and the absolute value of its WT which is obtained by retrieving the amplitude of the resultant absolute value of the WT along the dilation $d=\lambda z/x$. This dilation corresponds to the theoretical space-spatial frequency of the interference fringe. Here the Morlet wavelet defined by

$$g(x)=\exp(i2\pi x)\exp\left(-x^{2}/2\right) \tag{3}$$

was used as an analyzing wavelet. Evidently, the minima of the absolute value of the resultant WT output appear at the correct zero-crossing points of the chirp signal. By determining the frequencies of the chirp function at these minima, the size of the optical fiber can then be calculated.

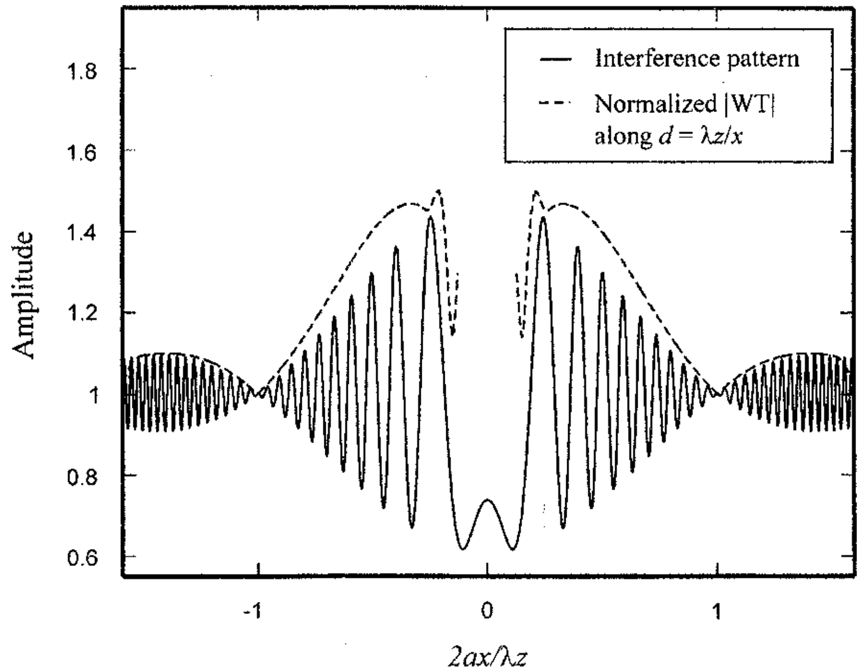

Figure 1. Simulated in-line hologram of the optical fiber and the normalized absolute value of its WT which is retrieved along the dilation $d = \lambda z/x$.

The errors in measurement of the optical fiber diameter by using the proposed methods are shown in Fig. 2. The holograms were recorded at the distance z from 12 cm to 25 cm. The errors in the measurement from the simulated holograms are represented by the circle sign, while the errors from the experimentally generated holograms are shown as the cross sign. It is found that the experimental results give a higher errors in comparison with the simulation. The reason may come from the imperfection of the interference pattern due to the speckle noise that overlapped the hologram. However, the errors of measurements by using the proposed method are still smaller than 1 percent because the proposed method employs only the frequency of the chirp signal at the minimum position of the envelope function for evaluating the object size. This frequency can be accurately determined by using the WT.

4. CONCLUSIONS

The absolute values of the WT have been employed to extract the particle size information from its in-line Fraunhofer holograms. By taking the absolute values of the resultant WT of the interference pattern, the frequency of the chirp signal at the minima positions can be determined. This is because the WT could extract the minima positions of the interference pattern. In the WT domain, these minima appear at the scales correspond to the frequencies of the interference fringe at the corresponding minima positions. The particle size which is inverse proportional to these frequencies can then be calculated. The small measurement error obtained from the simulation and experimental results verify the feasibility of the proposed method.

5. ACKNOWLEDGEMENT

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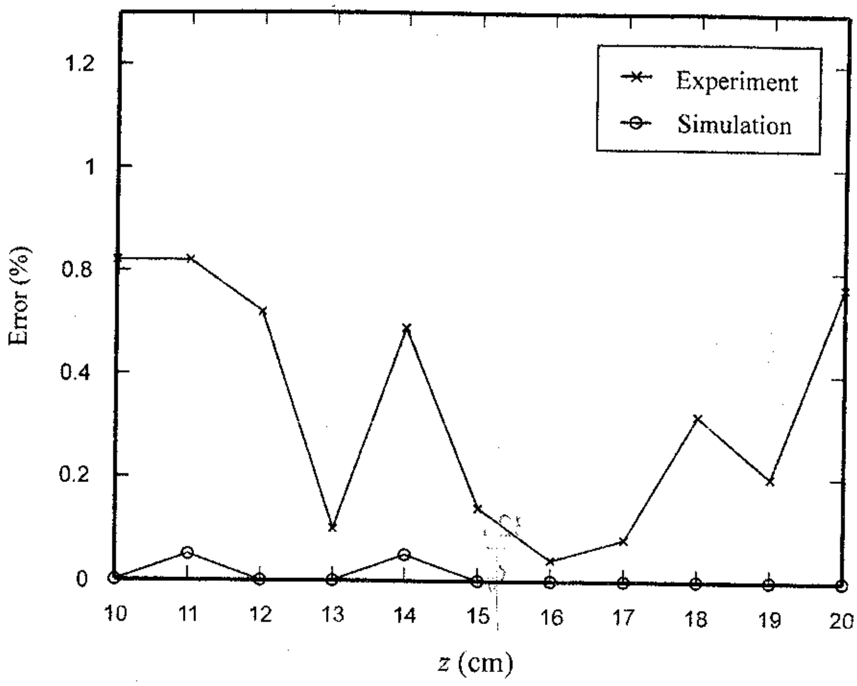

Figure 2. Errors in measurements of the optical fiber size from the simulated and experimentally generated holograms.

6. REFERENCES

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