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Invariants and invariant description of second-order ODEs with three infinitesimal symmetries. I

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Abstract

Lie's group classification of ODEs shows that the second-order equations can possess one, two, three or eight infinitesimal symmetries. The equations with eight symmetries and only these equations can be linearized by a change of variables. Lie showed that the latter equations are at most cubic in the first derivative and gave a convenient invariant description of all linearizable equations. Our aim is to provide a similar description of the equations with three symmetries. There are four different types of such equations. We present here the candidates for all four types. We give an invariant test for existence of three symmetries for one of these candidates.

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1. Introduction

According to Lie's classification [1] in the complex domain, any ordinary differential equation of the second order

$$y'' = f(x, y, y') \tag{1}$$

admitting a three-dimensional Lie algebra belongs to one of four distinctly different types. Each of these four types is obtained by a change of variables from the following canonical representatives (see, e.g., [2, Section 8.4]):

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