รหัสโครงการ SUT1-105-43-12-14

รายงานการวิจัย

ทฤษฎีสนามควอนตัม, ศึกษาด้านฟิสิกส์ทฤษฎี

(Quantum Field Theory and Theoretical Physics)

คณะผู้วิจัย

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ผลงานวิจัยเป็นความรับผิดชอบของหัวหน้าโครงการวิจัยแต่เพียงผู้เดียว

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ACKNOWLEDGMENT

This is to acknowledge with many thanks the Suranaree University of Technology Research Fund Administration for financial support which made this project possible and led to its successful completion.

บทคัดย่อ

ในการแผ่รังสีซินโครตรอน จำนวนเฉลี่ย <N> ของโฟตอนที่แผ่ออกมาในแต่ละรอบจะกำหนดอย่างแม่นตรงและชัดแจ้งซึ่งเกี่ยวข้องกับปริพันธ์มิติเดียว ในงานพิมพ์ต่าง ๆ ที่ผ่านมาเราเห็นว่าค่าที่พลังงานสูงมีความไม่แม่นและค่าความผิดพลาดสัมพัทธ์ มีค่าถึง 160 % ที่ความเร็ว 80% ของความเร็วแสง ซึ่งในรายงานนี้ได้มีการแสดงความสัมพันธ์ที่พลังงานสูงของ <N> ขึ้นมาใหม่ด้วย ได้มีการศึกษาการปลดปล่อยของการแผ่รังสีเชเรนโกฟที่อุณหภูมิอันตะในตัวกลางและได้คำนวณอย่างแม่นตรงถึงอันดับที่หนึ่งของค่าคงตัวโครงสร้างละเอียด ด้วยวิธีการหาปริพันธ์เชิงซ้อนเพื่อหลีกเลี่ยงการรวมตัวของตัวแผ่ไฟยน์แมนในรูปแบบอิงตัวแปรเสริมและใช้แทนการหาปริพันธ์เชิงซ้อน ซึ่งมีความเป็นไปได้ที่จะใช้ได้ที่อุณหภูมิศูนย์องศาสัมบูรณ์ เราได้แสดงให้เห็นว่าส่วนจินตภาพของพลังงานในตัวของอิเล็กตรอนซึ่งสอดคล้องกับเงื่อนไขขอบที่ถูกต้องเป็นผลให้ไม่ต้องการ contact term สำหรับการคำนวณนี้ และแง่มุมที่น่าพอใจที่เกิดขึ้นในพลศาสตร์ไฟฟ้าควอนตัมนี้ไม่ปรากฏในแบบแผนเดิมคือ ที่ความถี่สูง ๆ กำลังของการปลดปล่อยรังสีจะคัด-ออฟ (cut off) โดยอัตโนมัติ ซึ่งเป็นสิ่งที่สำคัญมาก ในการแก้ปัญหาเชิงควอนตัม

ได้มีการหาขอบเขตบนและขอบเขตล่างเกี่ยวกับพลังงานสถานะพื้นที่แม่นตรงของอะตอมที่เป็นกลางสำหรับค่าเลขอะตอม Z ที่เข้าสู่อนันต์ โดยใช้กรีนฟังก์ชันสำหรับศักย์ของอนุภาคเดี่ยว ค่าจำกัดของทั้งขอบเขตบนและล่างได้แสดงให้เห็นถึงความสอดคล้องกันกับพลังงานที่สถานะพื้นของทอมัส-เฟร์มี

ABSTRACT

An exact and remarkably simple one-dimensional integral expression is derived for the mean number $\langle N \rangle$ of photons emitted per revolution in synchrotron radiation for the first time. The familiar high-energy expression printed repeatedly in the literature is found to be inaccurate with a relative error of 160% for a speed of 80% of the speed of light. A new improved high-energy expression for $\langle N \rangle$ is given. The quantum electrodynamics of Cerenkov radiation at finite temperature is developed and computed exactly to first order in the fine-structure constant. By avoiding combining Feynman propagators in parametric form and using instead complex integration an exact evaluation, valid also at zero temperature, was possible. We show, in particular, that the imaginary part of the electron self-energy satisfies the correc boundary condition and no-contact term is needed in its evaluation. QED, unlike its classical counterpart, is shown to introduce automatically a cut-off for higher frequencies emphasizing the importance of the quantum treatment. Upper and lower bounds are derived for the exact ground-state energy of neutral atoms which for large Z both involve the limits of exact Green's functions with one-body potentials. The limits of both bounds are shown to coinside with the Thomas-Fermi ground-state energy.

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1. INTRODUCTION

The fascinating story of synchrotron radiation as a particular example of radiation theory emphasizing both the early theoretical and experimental developments is well documented in the literature (e.g., [1]). The monumental pioneering theoretical contribution of Schwinger [2,3] and its direct experimental impact has been particularly noted [1] to the extent that it was called in the early developments "Schwinger radiation". Although the main features of synchrotron radiation have been well known for a long time, there is certainly room for further developments and improvements. We have used an earlier integral for the power of radiation obtained by Schwinger [2,4,5] over fifty years ago to derive an exact expression for the mean number $\langle N \rangle$ of photons emitted per revolution with no approximations made. The derived result for $\langle N \rangle$ is a remarkably simple one-dimensional integral. We infer that the familiar high-energy expression [6,7] for $\langle N \rangle$ repeatedly printed in the literature is rather inaccurate and even for speeds of the charged particle with $\beta = 0.9$, 0.8 deviations from this expression are rather significant with large relative errors of 82%, 160%, respectively. In particular, our exact result for $\langle N \rangle$ is used to obtain a much-improved asymptotic high-energy expression for radiating particles.

The history of Cerenkov radiation is a long one with its discovery [8,9] in 1936 and its first theoretical explanation [10] in 1937. In recent years, the experimental studies [e.g., 11-14], the applications [cf.15,16] and the theoretical investigations for radiating particles [17-24] and strings [25] have been numerous. Although most of the recent theoretical studies were classical ones, many have also dealt at the quantum [e.g., 17, 21, 24, 25, 26] level since the earlier attempts [cf. 27-30] also at the quantum level. One of the clearest and most detailed quantum treatment of the problem we have found in the literature over the years is that of Schwinger et. al. [4,5].The latter deals with the full QED (quantum electrodynamics) to the first order in the fine-structure constant in an isotropic homogeneous medium at zero temperature. Unfortunately , due to the method of combining the denominators of the propagators in parametric form the resulting integrals turned out to be exceedingly complicated and approximations were necessarily made. This left, in particular, the higher order quantum correction undetermined. The ambiguity associated with the latter part of the quantum correction is well known. We have carried out a detailed study of the QED of Cerenkov radiation, exactly to first order in the fine-structure constant, with no further approximation at finite temperature [e.g., 31] for the first time and in the process obtained the full quantum correction to the spectrum, in an isotropic homogeneous medium. We use the method of complex integration directly on the electron self-energy without combining the denominators of the underlying propagators. This turns out to simplify the problem tremendously over the more conventional method of combining denominators of the propagators in parametric form. This allowed us to obtain a closed expression for the integral in question for the power. It was shown that the imaginary part of the electron self-energy satisfies the correct underlying boundary condition and no contact term was needed to be introduced. One of the most pleasing aspects of the QED treatment was found that, unlike its classical counterpart, it introduces automatically a cut-off for higher frequencies beyond which the power is necessarily zero.

A very remarkable property of atoms is that for large Z, the atomic number, the Thomas-Fermi [32,33] becomes exact [34-36]. Unfortunately, the very ingenious proofs of this beautiful result were somewhat complex. We have strived in developing a relatively easier derivation of this fundamental result by using, in the process, the Green's function corresponding to the Thomas-Fermi potential. Upper and lower bounds are derived for the ground-state energy of neutral atoms which for large Z both involve the limits of exact Green's functions with one-body potentials. The limits of both bounds are shown to coincide with the Thomas-Fermi ground-state energy.

From above we may summarize, in turn, by stating that the objectives of this project were to obtain exact results, for the first time, in two of the most celebrated forms of radiation - that is of synchrotron and Cerenkov radiation. Also establish, in a rather direct way, that the Thomas-Fermi theory does indeed provide the leading approximation for the exact ground-state energy for atoms in the small parameter 1/Z. The underlying hypotheses involved were the validity of electrodynamics at the classical and quantum level (versus QED) and that of ordinary quantum physics of multi-electron atoms. The usefulness of our exact results mentioned above is that they should meet future experiments since they were derived from well established theories. The scope of the research is limited to the usual idealistic conditions under which a theory is often derived. As, for example, a complete vacuum in the case of synchrotron radiation and that for the case of Cerenkov radiation, the medium may be described by introducing its permeability and permitivity as reacting to external sources. As mentioned above, however, the results are derived within well established theories.

The results obtained in the above investigations are published, respectively, in [38,39,40].

2. METHODOLOGY

We have used the well known expression for the power of radiation developed by Schwinger [2,4,5] almost fifty years ago to derive an exact expression for the mean number $\langle N \rangle$ of photons emitted per revolution in synchrotron radiation with no approximations made. The resulting expression for $\langle N \rangle$ is a remarkably simple one-dimensional integral. The basic idea in our derivation was to explicitly incorporate the correct boundary condition for $\langle N \rangle$ to be zero for $\beta \longrightarrow 0$ before carrying out the integrals involved in the theory. In carrying out our exact expression, to first-order in the fine-structure constant, of the power of Cerenkov radiation in quantum electrodynamics at finite temperature in isotropic homogeneous media we have relied on the following method. The expression for the power was written in terms of the self-energy of the electron. Instead of combining the denominators of the underlying propagators in parametric form, as done in the past, complex integration techniques were used. We have justified rigorously integrating over the complex domain by deriving lower bounds on singularities involved. The method of complex integration brings us into contact with studies of analytical properties of Feynman diagrams dealing with so-called pinching singularities [e.g.,37] In our large Z limit-study, the derivation rests on the fact that elementary scaling properties of the integrals of the Green's function allows one readily to consider this limit with no difficulty. The basic idea is that integrals of the Green's function for coincident space points involved in the analysis have particularly simple power law behaviour for large Z.

3. RESULTS OBTAINED

The exact expression for the mean number of photons emited per revolution in synchrotron radiation obtained in this work is given by

$$\langle N \rangle = 2\alpha\beta^2 \int_0^\infty \frac{dx}{x^2}\left[\frac{(\sin x/x)^2 - \cos(2x)}{1 - \beta^2(\sin x/x)^2}\right]$$

where $\alpha = e^2/\hbar c$ is the fine-structure constant, and β is the speed of the radiating charged particle measured relative to the speed of light. In particular, for high energetic particles, the exact expression given above yields

$$\langle N \rangle \simeq 5\pi\alpha/\sqrt{3(1-\beta^2)} + a_0\alpha$$

where $a_0 = -9.5580$. The relative errors of the above high-energy estimates are, respectively, 4.11 %, 1.34 %, 0.063 % for $\beta = 0.8, 0.9, 0.99$ in comparison with the result printed repeatedly in the literature [6,7] of 160 %, 82 %, 17 %, respectively.

The exact expression for the power of Cerenkov radiation, to first order in α, in quantum electrodynamics at finite temperature obtained in this work is given by

$$P_T(\omega) = \alpha\omega\beta\mu\left\{1 - (1/n\beta)^2\left[1 + (\omega/2E)(n^2-1)\right]^2 + (\omega^2/2E^2)(n^2-1)/\beta^2\right\} A_T(\omega)$$

where

$$A_T(\omega) = \frac{e^{\omega/kT}}{\left[e^{\omega/kT} - 1\right]}\left[\frac{\exp|E-\omega|/kT - \exp -\omega/kT}{(\exp|E-\omega|/kT + 1)}\right]$$

in units of $\hbar = 1$, $c = 1$, where ω is the energy of radiated photons, E is the total energy of the radiating charged particle, $n = \sqrt{\mu \kappa}$ is the index of refraction of the medium, μ and κ are, respectively, the permeability and permitivity of the isotropic homogeneous medium. T denotes the absolute temperature and k denotes Boltzmann's constant. QED, unlike its classical counterpart, provides naturally a cut-off for the frequencies $\omega \leq \omega_c$ beyond which the power is zero, where

$$\omega_c = (2(n\beta - 1)m)/((n^2-1)(1-\beta^2)^{1/2})$$

where m denotes the mass of the charged particle.

Finally, let E_Z denote the exact ground-state energy for a neutral atom with atomic number Z. We have shown, by using in the process basic scaling properties of one-body Green functions, that for large Z

$$E_{TF} \leqslant Z^{-7/3} E_Z \leqslant E_{TF}$$

where E_{TF} is the coefficient of $Z^{7/3}$ of the Thomas-Fermi ground-state energy. That is, $Z^{-7/3} E_Z \longrightarrow E_{TF}$ for $Z \longrightarrow \infty$.

4. SUMMARY AND CONCLUSIONS

The exact expression for the mean number of photons emitted per revolution in synchrotron radiation was derived and a corresponding high-energy estimate for the latter was also established. In particular, it was shown that the expression printed repeatedly in the literature gives relative errors of 160 % for a speed of 80 % of the speed of light and is rather inaccurate. An exact expression, in turn, for the power of radiation in quantum electrodynamics, to first-order in the fine-structure constant, was derived at finite temperature for the first time. In particular, the higher order quantum correction was also obtained which is valid at zero temperature as well. It was shown that the quantum correction introduces automatically a cut-off for higher frequencies of photons emitted unlike its classical counterpart. Finally, it was established, in a rather direct way, by using basic scaling properties of one-body Green functions that the Thomas-Fermi theory does indeed provide the leading approximation to the exact ground-state energy for atoms in the small parameter 1/Z.

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