MAX-MIN ANT SYSTEM FOR LOCATION-ROUTING PROBLEMS

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Abstract

This work introduces a modified meta-heuristic algorithm for solving Location- Routing Problems (LRP). It presents the most relevant steps towards the implementation of LRP, involving servicing a set of customers from a set of specific capacitated depots by using a set of identical vehicles. The objective of LRP is to minimize the total location and distribution costs. Since LRP is non-deterministic polynomial-time (NP) hard combinatorial problem, the heuristic is an appropriate approach to solve this problem. In this study, a heuristic based on the Max-Min Ant System (MMAS) is proposed and a 2–opt/ Move-Swap algorithm is applied. This approach aims to integrate 2 levels of decision making (location-routing) in a computationally efficient manner. Simulations are performed using problem instances available from literature. The results show that the modified MMAS performs efficiently in solving LRP.

Keywords: Location-Routing Problems, metaheuristic, max-min ant system, local search

Introduction

The concept of integrated logistics systems has given rise to a new management philosophy which aims to increase distribution efficiency. Such a concept recognizes the interdependence among the location of the facilities, the allocation of suppliers and customers to the facilities, and the vehicle route structure around the depots. The design of logistics systems requires a number of different types of strategic decisions. One of the higher level decisions that must be addressed involves the location of the facilities from which the activity of the system will be managed. Facilities must be located so as to minimize the operating costs of the system, so it is necessary to consider the facility location and distribution decision simultaneously. The combined location-routing model solved the joint problem of determining the optimal set of vehicle schedules and locations. Each location has a fixed operating cost and a capacity, and the traveling costs between any 2 points. The goal is to determine the number and locations of the facilities to be opened and design multiple routes from each selected location in such a way that each customer belongs to exactly 1 route, capacity constraints

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on the facilities are satisfied, and the total costs are minimized. There are a number of related papers involving combinations of LRP. Several different types of solution methods have been used for solving LRP.

These are exact algorithms and heuristics. Laporte and Norbert (1981), and Laporte *et al* (1983; 1986) developed a branch and bound algorithm that solves related sub-problems, adds upper bounds on variables, and branches on non-integer variables. They are able to solve some randomly generated symmetric instances of the Multi-Depot Vehicle Routing Problems (MDVRP) with as many as 25 problem nodes (including depots nodes). Laporte *et al*. (1988) solved some asymmetric MDVRP by performing a graph extension and then creating constrained assignment problems which they were able to solve through the branch and bound method. Using this method they solved problem instances with up 80 nodes, so long as the number of depots was small (2-3 depots), since location-allocation problems and the Vehicle Routing Problems (VRP) are NP-hard combinatorial problems which are difficult to solve by exact algorithms.

Clarke and Wright (1964), originated the saving and insertion heuristics to solve vehicle routing. These heuristics are efficient for forming good clusters for the customer nodes and depot nodes.

Chien (1993), proposed an approximate approach for the Multi-Depot Location Routing Problem (MDLRP), in which route length estimators are used in constructing vehicle routes.

Nagy and Salhi (1996) adopted the concept of the nested method to treat the routing element as a sub-problem within the larger problem of location. While still few in number, more papers have been written concerning heuristic approaches to the MDLRP. Gillet and

Johnson (1976) proposed an assignment sweep approach which is an extension of the sweep heuristic, and solved the MDLRP in 2 states: customers were first assigned to depots to compact and disjointed clusters and then independent single-depot VRP were solved using the sweep heuristic.

Raft (1982) presented a 2-phase heuristic that starts with a route assignment phase. After having estimated the number of vehicles needed, the algorithm constructs clusters of customers, each assigned to 1 vehicle. These clusters are not assigned any depot and are constructed to provide a small-expected length. In the next phase, each route is assigned to a depot, and then a 2-Opt exchange procedure is applied to each route.

Chao *et al*. (1993) provided a review of the previous heuristics in the operations research literature, and also introduced a new heuristic. The most important element in this new heuristic is the improvement procedure, which allows total distance to increase with the hope that a solution with an overall decrease may be found further along in the improvement process. The authors applied their new heuristic on data sets taken from the literature and found that the new heuristic yielded better solutions than were previously known.

Renaud *et al*. (1996) and Pathumnakul (1996) applied a Tabu Search (TS) heuristic to MDVRP. The algorithm contains 2 parts: construction of an initial solution by assigning customers to its nearest depots and then using a heuristic to find the best route selection and using a TS to improve the solution.

Ha (1998) presented a hybrid genetic approach for the MDVRP, that applied a genetic algorithm to cluster the MDVRP into VRP and then used a hybrid 2-Opt/Or Opt heuristic to solve single-depot VRP.

Madsen (1983) applied the TS, Tuzun and Burke (1999) presented a 2-phase TS for the MDLRP and compared the 2-phase algorithm with other heuristics, and Wu *et al.* (2002) applied simulated annealing, and threshold accepting and simulated annealing.

Sodsoon and Sindhuchao (2007) the MMAS and the Swap-Move/*2-Opt algorithm to solve the MMAS. The heuristic starts with customers assigned to each depot and vehicle routes constructed simultaneously using MMAS. After an ant colony has constructed all the routes completely the Swap-Move/

*2-Opt is applied to each route.

This paper focuses on the study of MMAS and solution improvement procedures (2–opt/ Move-Swap algorithm) for solving the LRP with multiple depots, multiple routes, homogeneous fleet (only one type of vehicles) and limited capacity of vehicles and the time window is not considered in this case. This paper is organized as follows: the model is formulated in section 2; in section 3, the MMAS and solution improvement are presented; computational experiments are discussed in section 4; and finally, conclusions are provided in section 5.

Location-Routing Problem

The location-routing problem was first defined by Perl and Daskin (1985). In this research, the formulation is closely related to Wu *et al*. (2002). The following information is known; number of candidate depots, number of customers, deterministic demand of each customer, vehicle capacity, dispatching cost for vehicles, and depot establishment cost. Each customer is served by exactly 1 vehicle. The total demand on each route is less than or equal to the capacity of the vehicle assigned to that route, and each route begins and ends at the same depot. Each vehicle is identical. The following indices, parameters, and decision variables are used in the mathematical model:

Notations and Decision Variables

I	set of all potential depot sites
J	set of all customers
K	set of all vehicles
N	number of customers
C_{ij}	distance between points i and j, $i, j \in I \cup J$
G_i	fixed costs of establishing depot i
F_k	fixed costs of using vehicle k
V_i	maximum throughput at depot i
d_j	demand of customer j
Q_k	capacity of vehicle (or route) k
$X_{ijk} = 1$	if point immediately precedes point j on route k $(i, j \in I \cup J)$; 0 = Otherwise
$y_i = 1$	if depot i is established; 0 otherwise
$Z_{ij} = 1$	if customer j is allocated to depot i; 0 = Otherwise
U_{lk}	auxiliary variable for sub-tour elimination constraints in route k

Mathematical Model

$$\text{Min: } \sum_{i \in I} G_i y_i + \sum_{i \in I \cup J} \sum_{j \in I \cup J} \sum_{k \in K} C_{ij} x_{ijk} + \sum_{k \in K} F_k \sum_{i \in I} \sum_{j \in J} x_{ijk} \tag{1}$$

Subject to

$$\sum_{k \in K} \sum_{i \in I \cup J} x_{ijk} \leq 1 \qquad j \in J \tag{2}$$

$$\sum_{j \in J} d_j \sum_{i \in I \cup J} x_{ijk} \leq Q_k \qquad k \in K \tag{3}$$

$$U_{lk} - U_{jk} + N x_{ljk} \leq N-1 \qquad l, j \in J,\ k \in K \tag{4}$$

$$\sum_{j \in I \cup J} x_{ijk} - \sum_{j \in I \cup J} x_{jik} = 0 \qquad k \in K,\ i \in I \cup J \tag{5}$$

$$\sum_{i \cup I} \sum_{j \in J} x_{ijk} \leq 1 \qquad k \in K \tag{6}$$

$$\sum_{j \in J} d_j z_{ij} - V_i y_i \leq 0 \qquad i \in I \tag{7}$$

$$-z_{ij} + \sum_{\text{uÎIÈJ}} (x_{iuk} + x_{ujk}) \leq 1 \quad i \in I,\ j \in J,\ k \in K \tag{8}$$

$$x_{ijk} = 0,1 \quad i \in I,\ j \in J,\ k \in K \tag{9}$$

$$y_i = 0,1 \ i \in I \tag{10}$$

$$z_{ij} = 0,1 \quad i \in I,\ j \in J \tag{11}$$

$$U_{lk} \geq 0, \quad l \in J,\ k \in K \tag{13}$$

The objective function minimizes the sum of the fixed depot-establishing cost, delivery cost, and dispatching cost for the vehicles assigned, respectively. Constraints Equation (2) require that each customer be assigned to a single route. Constraints Equation (3) are the capacity constraint set for vehicles. Constraints Equation (4) are the new sub-tour elimination constraint set. Flow conservation constraints are expressed in Equation (5).

Constraints Equation (6) assure that each route can be served at most once. Capacity constraints for the depots are given in Equation (7). A constraint (8) specifies that a customer can be assigned to a depot only if there is a route from that depot going through that customer. Constraint sets Equations (9), (10), and (11) are the binary requirements on the decision variables. The U_{lk}, auxiliary variables taking positive values are declared in Equation (12).

Heuristic for LRP

Let a set of customers and potential depots be presented by points on the plane. Each customer has a certain demand. The location has an installation cost of each site and the unitary cost of distribution. The vehicles routes and the potential depots have a certain capacity. The purpose of LRP is, then, to choose the depots that must be opened and to draw the routes from these depots to the customers, having an objective of minimizing the total location and distribution costs. The summarized algorithm is shown in Figure 1.

Grouping Phase

At this stage, finding a heuristic algorithm giving rapid approximations to the optimum seems to be more appropriate than well-developed methods which typically consume too much time. Hence, the grouping procedure assumes that all the potential depots are opened and each customer is assigned its nearest depot. The total demands of each group of customers do not exceed the depot capacity. We are dealing with the signal ratio which is the measurement of connection between customers and depot locations. The sector grouping algorithm is illustrated as follows:

1) Assume that all the candidate depots are opened

2) Assign customers to the nearest depots. The total demands of each group of customers for each depot must not be greater than the depot capacity.

3) Compute the signal ratio of each group. The signal ratio is the sum of the ratio between the demand load of a customer and the distance from that customer to a depot location according to Equations (2) and (3)

4) Sort the signal ratio (R_1, R_2,.....R_M) of all the depots in descending order for generating initial vehicle routes by MMAS

5) An ant constructs the routes

For this grouping algorithm, we rank the depot locations for opening while maintaining sufficient coverage to the customer area and each customer can be satisfied. The first location in the list is the best one to open first. Based on the concept of proximity between 2 elements, some measures of proximity among groups and the depot locations have been proposed: single linkage (nearest-neighbor). The distances (C_{ij}) between customers and depot locations are simply computed as Euclidean distances by Equation (14)

$$C_{ij}=\sqrt{(x_i-x_j)^2+(y_i-y_j)^2} \tag{14}$$

where, x_i, y_i are coordinates of node i and j, respectively. In Equation (15), the signal

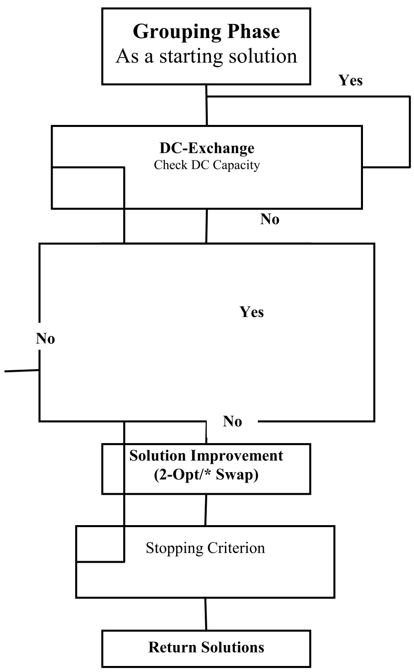


Figure 1. MMAS implemented

ratio at customer j and location i is assigned to the customer demand (d_j) divided by distances (C_{ij}):

$$\text{Ratio} : (R_1, R_2,....., R_M)$$

$$\sum_{j=1}^{n} (d_j/c_{ij}), \ \forall i, M \in I \qquad (15)$$

$$\text{subject to: } \sum_{j=1}^{n} d_j - V_i \leq 0 \qquad (16)$$

where, n is number of customers assigned to depot i , V_i is the maximum throughput at the depot i, and d_j is the demand of the customer j. The total demand of each group of customers must be less than or equal to the maximum throughput at the depot i.

Route Construction Phase

The MMAS algorithm is based on an Ant System (AS) algorithm developed by Dorigo and Gambardella (1997). In AS, m ants are initially positioned on n vertices according to an a priori assignment procedure. Each ant builds a tour by repeatedly applying a probabilistic nearest-neighbor heuristic. The MMAS introduced by Stützle and Hoos, (2000) is an improvement of the AS algorithm. In the MMAS, the pheromone trail is updated only on the global best and/or local best solution, instead of on solutions created by every ant, thus promoting a better exploitation of the search space. Another peculiarity is the inclusion of upper and lower bounds to the pheromone level τ_{min} and τ_{max} to help avoid stagnation. Initially the pheromone level of all trails is set to the upper bound in order to favor exploration. Therefore, the upper bound is initially chosen to construct a tour. Then, an ant modifies the pheromone level on the visited edges by applying a local updating rule.

Pheromone Trails Initialization

The pheromone level of each edge has lower and upper limits τ_{min} and τ_{max}. The initial pheromone, τ_0 , the upper limit, τ_{max}, and the lower limit, τ_{min} , are set as in Equation (17)

$$\tau_0 = \tau_{max}$$

$$\tau_{max} = M$$

$$\tau_{min} = \tau_{max}/(2*(N+M)) \qquad (17)$$

where, M is the number of depots and is the number of nodes in the graph, respectively.

Tour Construction

In this research, we adopt the concept similar to the elitist ant or ranked ant of Bullnheimer *et al.* (1999) and Dang (2003) of ant colonies constructing vehicle routes by alternating the motion of each ant from each depot. An ant selects the next customer to be served, compatible with capacity constraints. We used the number of ant colonies equal to the number of depots to construct routes. Each ant is put at a depot and each ant will choose the next nodes to move from the present node i to the next node j according to the state transition rule given by Equation (18).

$$P_{ij} = \begin{cases} \dfrac{[\eta_{ij}]^{\beta}[\tau_{ij}]}{\sum_{j \in U_k}[\eta_{ij}]^{\beta}[\tau_{ij}]} & \text{if } (i,j) \in \text{candidate list} \\ 0 & \text{otherwise} \end{cases} \qquad (18)$$

where, U_k is the set of nodes that remain to be visited by an ant positioned on node i,τ_{ij} is pheromone level on edges (i,j), and η_{ij} is the inverse of the length of edges (i,j). Thus, $\eta_{ij} = 1/d_{ij}$ where d_{ij} denoted the distance between nodes i and j, and β is the parameter that determines the relative influence of the pheromone. We used $2 \leq \beta \leq 5$ in the MMAS algorithm.

Local Pheromone Trail Update

Additionally to the global updating rule, in MMAS the ants use a local update rule that they apply immediately after having crossed an arc during the tour construction:

$$\tau_{ij} = (1-\xi).\tau_{ij} + \xi.\tau_0 \qquad (19)$$

where, ξ; $0 < \xi < 1$ and τ_0 are 2 parameters to the MMAS algorithm. In this way the exploration of not yet visited arcs is increased. The value of τ_0 is set to be the same as the initial value for the pheromone trails. Experimentally, a good value for ξ was found to be 0.1, while a good value for τ_0 was found to be $1/n.L^m$, where n is the number of cities in the LRP instance and L^m is the length of the nearest-neighbor tour.

Update of Pheromone Trails

The MMAS to update pheromone the trails includes iteration-best and global-best solutions to avoid search stagnation. The allowed range of the pheromone trails strength is limited to the interval $[\tau_{max}, \tau_{min}]$ and τ_{ij} is $\tau_{min} \leq \tau_{ij} \leq \tau_{min}$. The pheromone trails are initialized to the upper trail limits. After all ants have constructed solutions, the pheromone trails are updated according to Equation (20).

$$\tau_{ij}(t+1) \leftarrow (1\text{-}\rho)\tau_{ij}(t)+\Delta\tau_{ij}^{best} \quad (20)$$

where ρ is a parameter called the evaporation coefficient, $0 \leq \rho \leq 1$ and $\Delta\tau_{ij}^{best} = 1/C^{best}$ where t is scheduled for the fre quency and C^{best} is the best so far tour. The ant which is allowed to add pheromone trails may construct an iteration-best tour and global-best tour. All edges (i,j) belonging to the so far best solution (objective value) are considered to increase the intensity of pheromone trails by an amount $\Delta\tau_{ij}^{best}$. If edges (i,j) do not belong to the so far best solution, the intensity of the pheromone will be reduced. Heuristic approaches to the tour obtained by ants can be classified as tour constructive heuristics. Tour constructive heuristics usually start by selecting randomly a customer point and building the feasible solution piece by piece by adding new customers' points chosen according to the selected heuristic rule. Thus, the complete algorithm together with the flow for the method of study is summarized as shown in Figure 2.

Route Construction

In the stage of initialization, there are steps to generate a feasible initial solution. In order to apply MMAS to solve LRP, a modified MMAS is proposed. Each ant builds the solution by the state transition rule. An ant selects the next customer to be served, compatible with capacity constraints and limited route length constraints. This heuristic assigns customers to each depot and constructs vehicle routes simultaneously. If the accumulative loading of the ant exceeds the capacity constraints, it will return to the depot. This is called a complete vehicle route. Thus, we will focus on our heuristic to improve the original algorithm according to Equation (5). The section of route constructing is illustrated in Figures 3 and 4.

Initialize

Loop //Each loop called an iteration

  Each ant is placed on a starting customer's point

  Loop //Each loop called a step

    Each ant constructs a solution by applying a state transition rule and a local pheromone updating

  until all ants have constructed a complete solution.

  Each ant is brought to a local minimum by a tour

  improvement heuristic

  A global pheromone updating rule is applied.

until stopping criteria are met

Figure 2. Pseudocode of MMAS

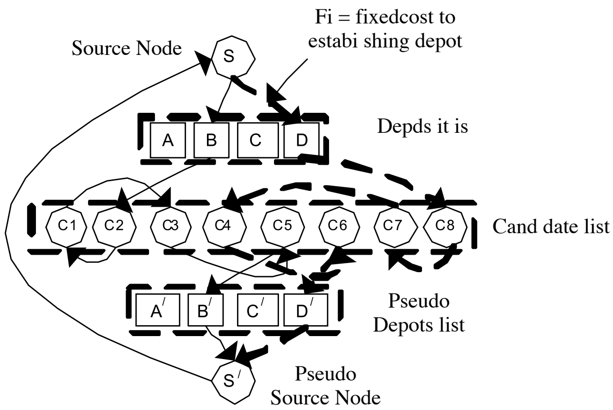


Figure 3. An ant constructing routes

Improvement Solutions

After an ant has constructed its solution, we apply a local search algorithm to improve the solution quality, called the solutions improvement procedure. In particular, we apply Swap, Move operator, and 2-Opt to the solution. To understand the Local Search algorithm clearly, an example with 2 depots and 12 customers is considered, which is illustrated in Figures 5, 6, 7, and 8. The Local Search aims to improve the solution by exchanging a customer *i* of 1 route with a customer of *j* another route. The Swap operator aims to improve the solution by exchanging a customer *i* of 1 route with a customer *j* of another route. The Move-Operators may interchange a customer within the same route and intra-route improvement or within the same depot and intra-depot improvement. Additionally, we also swap a customer from 1 route to another route, that is, inter route improvement or from 1 depot to another depot improvement. Given a solution S to an instance of the optimization problem, if there is no better solution the algorithm terminates with the current solution as the local optimum.

Numerical Analysis

In this section, we present the development of the MMAS program for solving LRP by using Microsoft Visual C++ 6.0 and executed on a PC with a 3.07 GHz Intel Pentium[R] 4 CPU and 224 MB of RAM. The customers' locations are in the form of a Cartesian coordinate where each point appears uniquely in a plane through 2 numbers, called the x-coordinate and y-coordinate while demands of customers which are known are recorded in a range form. Since there is no open source data for LRP, the data is adopted from a set of data modified from the well known problem in Wu *et al.* (2002) and Wang (2005) as in Table 1.

The numerical analysis was performed on set of benchmark problems that consists of 3 instances containing between 12 and 85 customers and 2 -15 depots. All instances have data of constrained capacity of the depot (unit), capacity of vehicle (unit), fixed cost for establishing the depot ($), transportation cost ($/mile), and fixed cost for using vehicles ($/ unit). Table 1 contains the data for the 3 problem instances.

Parameter Testing

To see the effect of the parameters of the MMAS on the distance traveled, an experimental design was carried out in the case studies. To reach reliable fixed conclusions, an ANOVA statistical test was applied and used to study the relationship that exists between a dependent

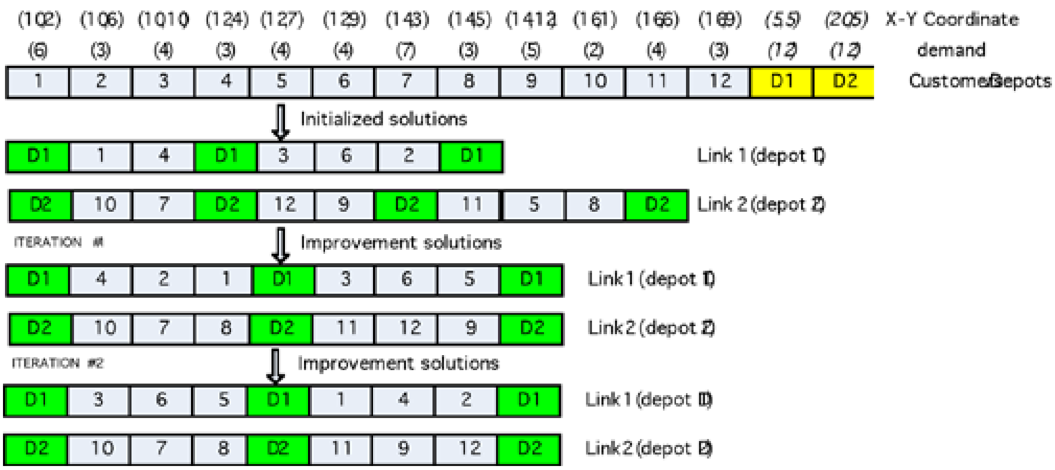


Figure 4. MMAS constructing routes

variable (variables response) and 1 or more independent variables (called factors) and thus, to learn whether the difference in the response depending on the variation of 1 of the factors was a random result. The theoretic F distribution value was calculated for a significance level of 95%. In these case problems, the LRP have a very limited capacity; we used, $\beta = 3.5$, $\rho = 0.99$ and n=300 as shown in Figure 9. We have used those values of β and ρ to run the experiments. The results of a good solution in P03 are shown in Figure 10.

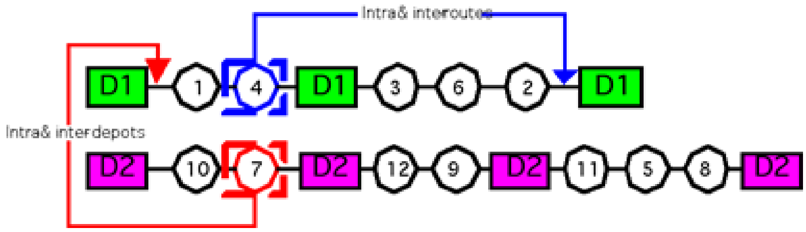


Figure 5. Swap operators

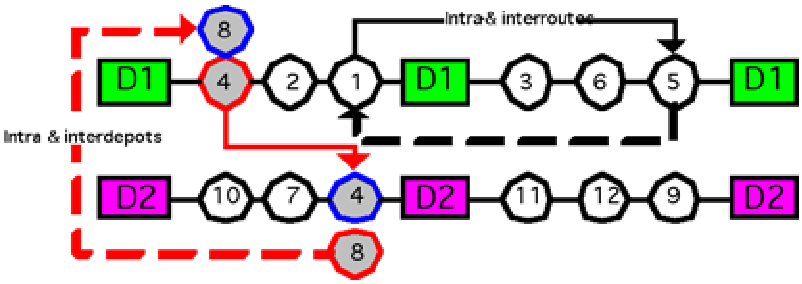


Figure 6. Move operators

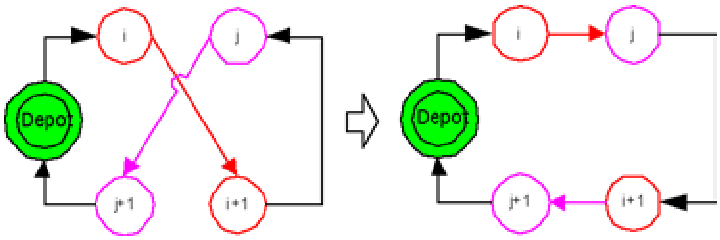
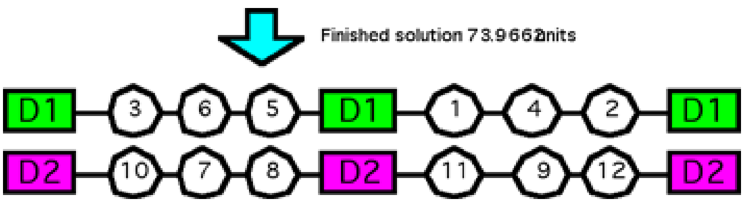


Figure 7. 2-opt algorithm



Figrue 8. Best solutions

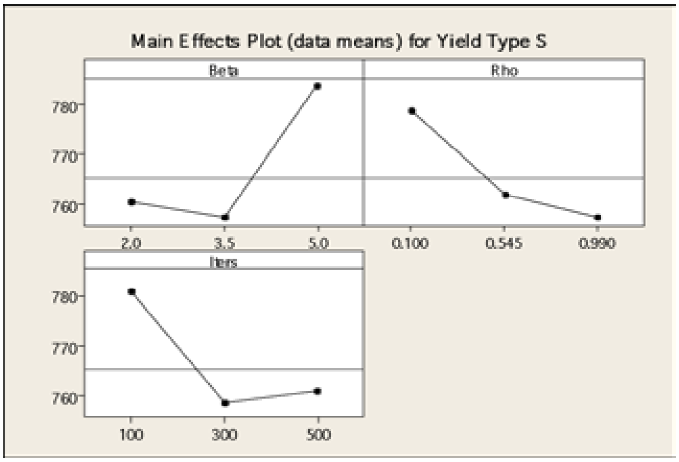


Figure 9. Effect of the parameters of MMAS

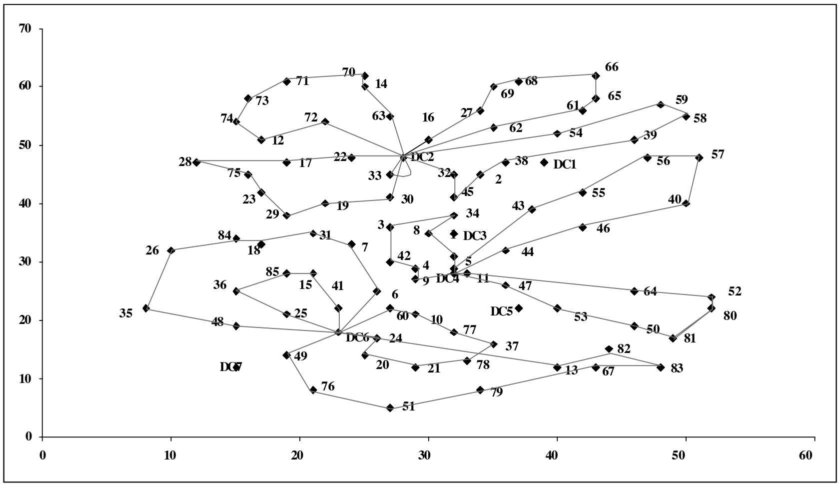


Figure 10. Computational tested in P03

Table 1. Location-routing problem instances

Name instances	References	Number of depot	Number of customer	Vehicle capacity	Depot capacity	Fixed cost to open depot ($)	Transportation cost ($/per miles)	Fixed cost for using vehicle ($/Unit)
P01:12x2	Perl and Daskin (1985)	2	12	140	280.0	100.0	0.75	0.74
P02:55x15	Perl and Daskin (1985)	15	55	120	550.0	240.0	1.00	0.74
P03:85x7	Perl and Daskin (1985)	7	85	160	850.0	372.0	1.00	0.74

Computational Tested

From the parameters testing section, we choose $\beta = 5$, and $\rho = 0.98$ to test the MMAS with a Local Search to solve the LRP. We test our heuristic with 3 problems from literature and to investigate the effectiveness of this heuristic the number of iterations is n*2 for each problem and is solved 5 times. The results are provided in Tables 2-4 in terms of the distance of route, and load of route value after the solution improvement phases. It can be observed from Table 5 that the proposed method is able to find the optimal solution for test problem P01 in only 0.063s. For problem

Table 2. Best solution results for the test P01

					103.97: Total distances
					355.58: Total cost
Depots	**Distances of route**	**Load of route**	**Sequence of customers**	**Customers served**	**Run Time CPU (sec):**
1	44.344	140	13 7 3 2 1 6 8 9 13	12	0.063s
	59.6327	100	13 10 12 11 5 4 13		

Problem: P01, customer=12, Depot=2, Capacity Depot= 280, Vehicle capacity=140

Table 3. Best solution results for the test P02

					4,100.05: Total distances
					5,634.05: Total cost
Depots	**Distances of route x 10**	**Load of route**	**Sequence of customers**	**Customers served**	**Run Time CPU (sec):**
2	56.3764	120	57 14 27 54 39 38 16 57		
	44.4048	120	57 12 28 23 19 17 22 57	12	
					51.656s
10	6.0645	40	65 13 11 65		
	51.2828	120	65 44 46 40 55 43 8 65		
	69.4971	120	65 52 50 53 47 37 10 65	26	
	16.3891	120	65 5 1 2 42 4 9 65		
	40.434	120	65 34 45 32 33 30 3 65		
12	30.7321	100	67 15 7 31 29 18 67		
	39.4966	120	67 36 26 24 35 48 25 67	17	
	55.3281	120	67 49 51 21 20 41 6 67		

Problem: P02, customer=55, Depot=15, Capacity Depot= 550.0, Vehicle capacity=120

P02 in Table 6, the proposed method outperformed both Perl and Daskin's (1985) and Wang *et al*'s (2005) results in terms of total costs. Notice that Hansen *et al*'s (1994) method results in less distance because 1 more distribution center (DC) is established than in the proposed method. The proposed method still provides a better solution than Perl and Daskin's (1985) and Wu *et al*'s (2002) studies in test problem P03 as shown in Table 7; however, Hansen *et al*'s (1994) method gives even fewer costs and distances than the proposed method. We summarize the computational results that include the best known solutions,

Table 4. Best solution results for the test P03

					5,266.4: Total distances 7,640.4: Total cost
Depots	**Distances of route x 10**	**Load of route**	**Sequence of customers**	**Customers served**	**Run Time CPU (sec):**
2	46.8265	160	87 63 14 70 71 73 74 12 72 87		
	6.32456	20	87 33 87	33	
	43.9812	160	87 30 19 29 23 75 28 17 22 87		
	44.9048	160	87 16 27 69 68 66 65 61 62 87		
	57.6393	160	87 54 59 58 39 38 2 45 32 87		
					156.52s
4	48.6981	160	89 47 53 50 81 80 52 64 11 89		
	59.6847	160	89 44 46 40 57 56 55 43 1 89	23	
	29.7136	140	89 9 4 42 3 34 8 5 89		
6	34.2665	160	91 24 20 21 78 37 77 10 60 91		
	69.1820	160	91 13 82 83 67 79 51 76 49 91	29	
	27.9814	160	91 25 36 85 15 41 91		
	57.4370	160	91 48 35 26 84 18 31 7 6 91		

Problem: P03, customer=85, Depot=7, Capacity Depot= 850.0, Vehicle capacity=160

Table 5. Results for comparison of test problem P01

Methods	Depot established	Number of routes	Sum total distances	Total costs	CPU run time (Sec)
Optimum	**1**	**2**	**103.97**	**355.58**	**N/A**
Perl and Daskin (1985)	1	2	103.97	355.58	N/A
Hansen *et al*. (1994)	1	2	103.97	355.58	N/A
Wu *et al*. (2002)	1	2	103.97	355.58	N/A
Wang- *et al*. (2005)	1	2	103.97	355.58	N/A
Proposed Method	1	2	103.97	355.58	0.063s

Table 6. Comparison of test problem P02

Methods	Depot established	Number of routes	Sum total distances	Total costs	CPU run time (Sec)
Perl and Daskin (1985)	2, 10, 12	10	4261.32	5795.62	N/A
Hansen *et al*. (1994)	2, 7, 12,13	10	3843.67	5617.67	N/A
Wu *et al*. (2002)	5, 10, 12	10	3998.28	5532.28	N/A
Wang- *et al*. (2005)	2, 10, 12	10	4198.72	5732.13	N/A
Proposed Method	2,10,12	10	4100.05	5634.05	51.656s

Table 7. Comparison of test problem P03

Methods	Depot established	Number of routes	Sum total distances	Total costs	CPU run time (Sec)
Perl and Daskin (1985)	2, 4, 5	11	5415.96	7789.96	N/A
Hansen *et al*. (1994)	2, 4, 6	11	5177.61	7551.61	N/A
Wu *et al*. (2002)	2, 4, 6	12	5407.21	7781.21	N/A
Wang- *et al*. (2005)	2, 4, 6	11	5265.69	7639.46	N/A
Proposed Method	2, 4, 6	12	5266.40	7,640.40	156.52

Table 8. Relative percentage deviation of total traveled distance and total cost

Instances	Researcher	Total Distances	(%)RPD	Total Cost	(%)RPD
P01	Perl and Daskin (1985)	*103.97**	0.00	*355.58**	0.00
	Hansen *et al*. (1994)	103.97	0.00	355.58	0.00
	Wu *et al*. (2002)	103.97	0.00	355.58	0.00
	Wang- *et al*. (2005)	103.97	0.00	355.58	0.00
	Proposed Method	103.97	0.00	355.58	0.00
P02	Perl and Daskin (1985)	4261.32	10.87	5795.62	4.76
	Hansen *et al*. (1994)	*3843.67**	0.00	5617.67	1.54
	Wu *et al*. (2002)	3998.28	4.02	5532.28	0.00
	Wang- *et al*. (2005)	4198.72	9.24	*5732.13**	3.49
	Proposed Method	4100.05	6.67	5634.05	1.84
P03	Perl and Daskin (1985)	4261.32	10.87	7789.96	3.16
	Hansen *et al*. (1994)	3843.67	0.00	7551.61	0.00
	Wu *et al*. (2002)	3998.28	4.02	7781.21	3.04
	Wang- *et al*. (2005)	4198.72	9.24	7639.46	1.16
	Proposed Method	4100.05	6.67	7713.54	2.14

In that table, the following notation is used:

BT = solution of algorithm

BKS = the best known solution from heuristic algorithm

(%)RPD = ((BT- Obj.-BKS)/BKS)*100%

literature, solutions obtained by MMAS and the deviations of total traveled distance, and the total cost from the best known solutions (Relative Percentage Deviation (RPD) in Tables 8.

Conclusions

In this paper, we have proposed a MMAS with a Local Search for LRP. The proposed algorithm can obtain the solution of LRP within a reasonable time. It can be used for redesigning the logistics network as well as improving the planning of the distribution network. The proposed method was compared with other heuristic approaches on 3 test problems and the results indicate that this method performs well in terms of the solution quality and run time consumed. For further study, we may develop a hybrid metaheuristic scheme that combines the strength of trajectory methods like Very-Large Scale Neighborhood Search and Ant Colony Optimization in order to increase the effectiveness in getting the optimal solution

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